Differential bevel gears

Strength calculation

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Contents

1 Introduction

This instruction explains about strength calculation and rating methods for forged bevel gears used in Differential casings, with KISSsoft.
For other topics in differential bevel gears, please refer to the following instructions:

Document name	Content
KISSsoft-anl-068-E-3D Geometry of Spiral Bevel Gear_rev6	Settings in 3D modelling, Parasolid
KISSsoft-anl-101-EN-Bevel Differential	Sizing of tip and root alteration

1.1 Design of differential gearboxes

Forged bevel gears are typically found in differential gearboxes of vehicles, as the forging process allows an economical manufacturing process for the high number of workpieces. The driving gears are typically called pinions, the output gears are called side gears. These nominations are used in this instruction too. The picture below shows a two-pinion design of a differential bevel gear (Source: http://www.metodolog.ru/triz-journal/archives/2008/09/03/index.html). Also common are four-pinion designs (for heavy duty applications) and sometimes also three-pinion designs. KISSsoft uses the term ‘strand’ for the torque flow of one pinion to two side gears (see below, static calculation).

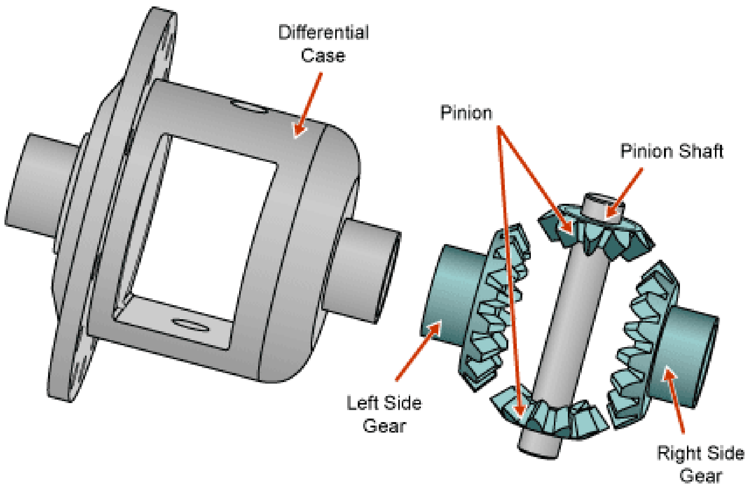


Figure 1. Principal sketch of two-pinion design with two pinions and two side wheels

1.2 Overview of rating methods

For strength rating, the forged differential bevel gears are basically handled as machine elements and hence, the rating methods for bevel gears can be applied. However, due the forging process, at heel and toe side there are webbings which result in a strengthening effect for root capacity. This is not considered in the rating standards like ISO 10300, AGMA 2003, DIN 3991 etc. and leads to low safety numbers as the calculation is too conservative (neglecting the positive effect of the webbing). Still, the standard can be used for estimation and comparisons between the design variants or competitors design. For a final rating, the analysis by FE methods is recommended.

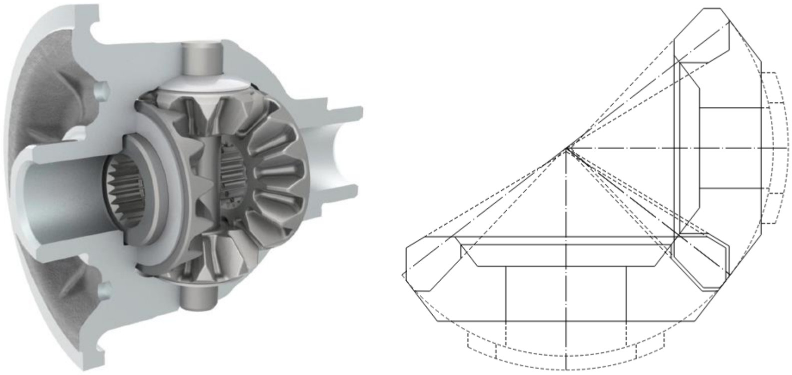


Figure 2. Forged differential bevel gears with webbings at toe and heel side (left, source: GKN) and representation in KISSsoft (right)

The goal in the design phase of differential bevel gears is to find the optimal tooth proportions and cone angles, to get highest load capacity, but also highest contact ratio, etc. The forging process allows the modification of root and face cone angles, which gives a high number of possible geometrical solutions.

Each of these solutions has to be checked for proper gear geometry, such as avoiding undercut and pointed tip. As in this phase, the rating by FE methods is not available, or very slow and hence inefficient, the rating by standards is a fast and good approach.

2 Strength calculation methods in KISSsoft

In KISSsoft, two rating approaches are available, the static rating and the fatigue rating. For the static rating, a direct comparison between calculated stress and yield / ultimate strength is used. For the fatigue rating, different standardized methods (along DIN, ISO and AGMA standards) are used. Also, the software handling is explained.

2.1 Torque split between pinions and side gears

For the strength calculation, it is important to consider the correct torque per gear mesh, as the rating calculation considers always one mesh of pinion and side gear. As shown above, in a differential casing there are always several pinions, but two side gears. So, the acting torque from the differential casing needs to be divided by the number of pinions. Each pinion has two meshes, what divides the torque by factor two additionally.

For the KISSsoft static calculation, the torque from the differential casing (T_diff, see below) is converted by the number of strands into the torque for the gear mesh. For the rating by ISO standard, the user needs to convert the differential casing torque into the gear mesh torque manually. With the system software KISSsys, the operating gear mesh torque for the bevel gear calculation is directly calculated within KISSsys.

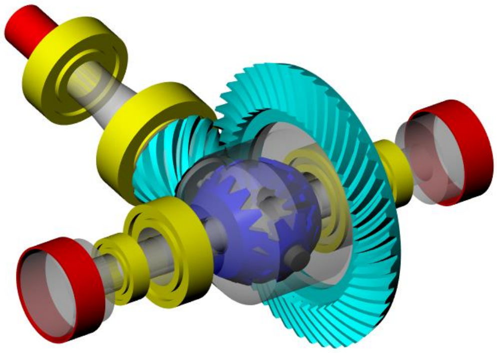


Figure 3. KISSsys model of differential casing, and calculation of gear mesh torque

2.2 Static calculation

2.2.1 Calculation approach

When the vehicle is driving in straight direction, the speed of the left and right axle shafts is identical. This leads to a non-rotating condition for the differential bevel gears and hence a static application of the gears. As the ISO rating method is based on tests with gears in rotating condition, the approach of the ISO is not suitable for the static condition.

The static calculation in KISSsoft provides a safety number for root bending load only. Therefore, the highest root stress is calculated, based on the highest acting force and lever arm. The root bending stress is compared with the static material properties as tensile strength and yield point.

2.2.2 Operating data

In KISSsoft, when choosing the calculation method ‘Differential, static calculation’, the torque is shown for the differential casing. The ‘reference gear’ input field becomes inactive and hence is not valid for pinion or side gear.

Figure 4. Operating data in KISSsoft tab ‘rating’

In the KISSsoft report, the operating data are shown for the differential casing (n_diff, T_diff, P_diff).

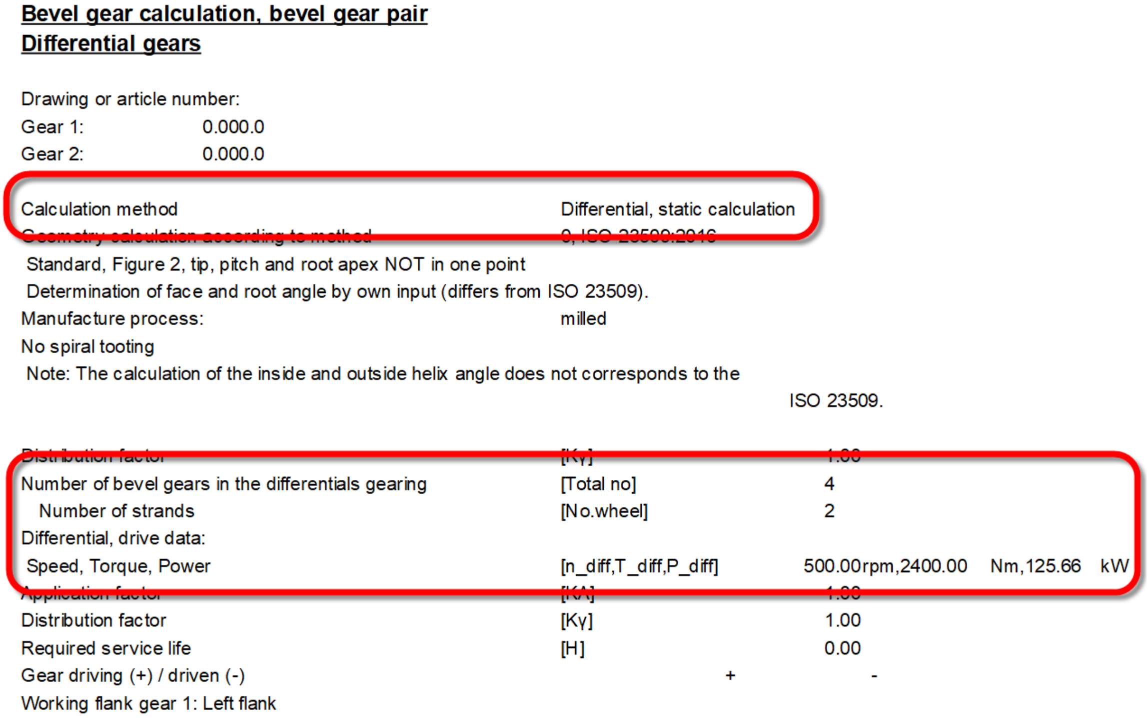


Figure 5. Operating data in the KISSsoft report

2.2.3 Gear mesh torque by number of strands

For the calculation method ‘Differential, static calculation’, the differential casing torque T_diff is divided by the number of strands into the torque per strand. One strand means the torque flow from one pinion to two side gears. In case of a differential casing in a ‘two-pinion design’, the number of strands is 2. Consequently, for a three-pinion design, the number of strands is 3, etc. This is to be entered under ‘details’ in the tab ‘rating’.

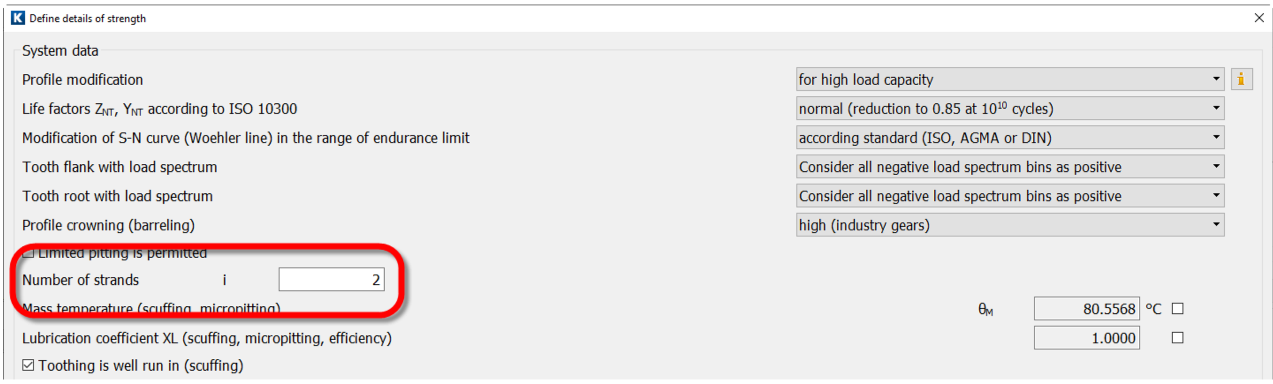


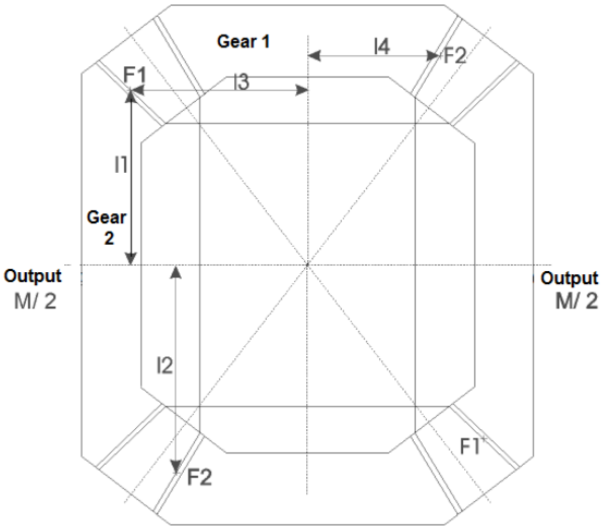
Figure 6. Input of number of strands

2.2.4 Determination of highest circumferential force

The torque of the differential casing is calculated by the following equation (see the KISSsoft report, section ‘general influence factors’):

T_diff = number of strands *(Ft1*l1 + Ft2*l2)

The differential torque T_diff is represented by the circumferential forces Ft1, Ft2 and the lever arms at contact of root l1 and at contact of tip l2. The KISSsoft manual shows the definition of the length and the calculation (https://www.kisssoft.ch/Manual/en/8581.htm).



$$M = 2 \cdot F_1 \cdot l_1 + 2 \cdot F_2 \cdot l_2$$

$$F_1 = F_2 \cdot \frac{l_4}{l_3}$$

$$l_1 = \sqrt{R_{m1}^2 - \left(\frac{d_{m1}}{2}\right)^2} - 0.5 \cdot \tan(\delta_1) \cdot (d_{am1} - d_{m1})$$

$$l_2 = \frac{d_{am2}}{2}$$

$$l_3 = \frac{d_{am1}}{2}$$

$$l_4 = \sqrt{R_{m2}^2 - \left(\frac{d_{m2}}{2}\right)^2} - 0.5 \cdot \tan(\delta_2) \cdot (d_{am2} - d_{m2})$$

Figure 7. Definition of lengths l1 and l2 for the static calculation of differential bevel gears

Based on the equation, the maximum circumferential force between F1 and F2 is determined and used for the rating of the strength. In the KISSsoft report, the maximum circumferential force is shown in the section of ‘general influence factors’.

General influence factors

Calculation of circumferential forces considering the contacts at root and tip			
Drive gear, lever arm at contact of root (mm)	[l1]	24.8	
Drive gear, lever arm at contact of tip (mm)	[l2]	29.7	
Circumferential force Ft1, Ft2 (N)	[Ft1,Ft2]	18522.6	,24952.2
Formula: 2 *(Ft1*l1 + Ft2*l2) = T_diff			
		------- Gear 1	-------- Gear 2 -------
Nominal circum. force at pitch circle (N)	[Fmt]	24952.2	24952.2
	Drive side		
Axial force (N)	[Fa]	6773.5	8805.6
Radial force (N)	[Fr]	8805.6	6773.5
Normal force (N)	[Fnorm]	27313.5	27313.5
Axial force (%)	[Fa/Ft]	27.146	35.290
Radial force (%)	[Fr/Ft]	35.290	27.146

Figure 8. Determination of highest circumferential force

2.2.5 Calculation of static safety

The static safety is calculated using the material properties yield point [σs] and tensile strength [σb]. In the KISSsoft, both the safety against plastic deformation and the safety for tensile stress are shown. Additionally, the safety against plastic deformation considering the stress correction factor YS is shown. In the KISSsoft result window, only the safety numbers without the stress correction factor YS are shown.

Yield point (N/mm²)	[σs]	850.00	850.00
Tensile strength (N/mm²)	[σb]	1200.00	1200.00
Safety against plastic deformation	[Ss=σs/σF]	0.80	0.78
Safety for tensile stress	[Sb=σb/σF]	1.13	1.10
Supplemented by stress correction factor YS:			
Calculation formula:	σF0 = Fmt / beff / mn * YF * YS * Yε * Yβ * YK		
Stress correction factor	[YS]	1.75	1.67
Safety against plastic deformation	[Ss=σs/σF]	0.46	0.47

Figure 9. Calculation approach and safety values in the KISSsoft report

Further literature for the calculation approach for static safety can be requested from KISSsoft support. Several publications in this topic were done from KISSsoft (e.g. 'static strength calculation on gears' from Dr. Kissling, dated 2013).

2.2.6 Safety number

The minimum required safety number differs strongly from the usual numbers such as the recommend value of 1,5 for straight bevel gears (see ISO 10300). As the webbing achieve a huge strengthening effect, the numbers typically are much lower than 1. Safety numbers are even possible in the area of 0.3 to 0.5. Therefore, a final test bench run is strongly recommended to verify the capacity of the differential bevel gears.

2.3 Calculation of load capacity of bevel gears, ISO 10300

2.3.1 Calculation approach

The ISO 10300 is an up-to-date and widely accepted standard for rating of bevel and hypoid gears. In case of the differential bevel gears, the standard applicable for the condition, when the vehicle is cornering. The ISO standard is also capable to rate additional failures such as pitting, scuffing, etc.

2.3.2 Operating data

When choosing the rating method 'ISO 10300' (the latest edition 2014 is recommended), the operating data are required for one gear mesh between pinion and side gear. This means, the torque of the differential casing is to be divided manually by the number of strands (between 2 and 4) and by the number of side gears (always 2).

Let's look at a practical sample. The torque of a differential casing for a two-pinion design is 2400 Nm. As the differential casing torque goes into both shaft axles left and right, the torque per side gear is 1200 Nm. With a two-pinion design, each of the side gears has two meshes due to the two pinions. So, the final torque per mesh is 600 Nm. This is to be entered into KISSsoft, under the tab 'rating'. The speed is to be entered according to the design specification.

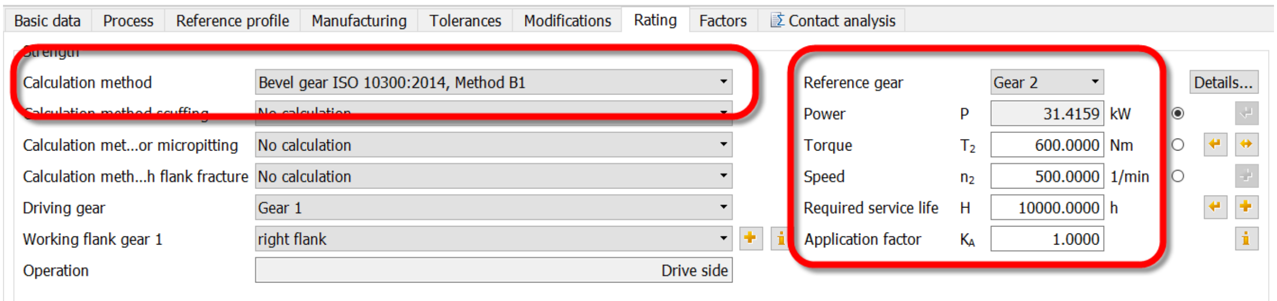


Figure 10. Operating data in KISSsoft tab 'rating'

In the KISSsoft report, the operating data are shown for the gears 1 (pinion) and 2 (side gear).

Bevel gear calculation, bevel gear pair

Drawing or article number:
Gear 1: 0.000.0
Gear 2: 0.000.0

Calculation method	Bevel gear ISO 10300:2014, Method B1 Drive side		
Geometry calculation according to method	0, ISO 23509:2016		

Standard, Figure 2, tip, pitch and root apex NOT in one point
Determination of face and root angle by own input (differs from ISO 23509).

Manufacture process: milled

No spiral tooting

Note: The calculation of the inside and outside helix angle does not corresponds to the ISO 23509.

		------- Gear 1 --------	Gear 2 -------
Power (kW)	[P]	31.416	
Speed (1/min)	[n]	650.0	500.0
Rotation direction, gear 1, viewed on cone tip:	right		
Torque (Nm)	[T]	461.5	600.0
Application factor	[KA]	1.00	
Distribution factor	[Kγ]	1.00	
Required service life	[H]	10000.00	
Gear driving (+) / driven (-)		+	-

Working flank gear 1: Right flank

Figure 11. Operating data in the KISSsoft report

2.3.3 Alternating bending factor

The alternating bending factor considers the alternating load on the tooth due to lower permissible stresses (haigh diagram). When the vehicle is cornering, the speeds between the left and right axle are different. This results in a relative motion between pinion and side gear. Looking at the pinion, it has alternating load due to the two side gears in contact. In contrary to the pinion, the bending load for the side gear is on the same flank.

The ISO 10300 does not have an alternating bending factor, so, the approach from ISO 6336 can be taken instead. In KISSsoft, the settings for the alternating bending factor is in the tab ‘factors’. For nominal load situation, the alternating bending factor for the pinion is 0.7, the alternating bending factor for the side gear is 1. However, the user needs to check his current load cases carefully and consider about the suitable alternating bending factors specifically.

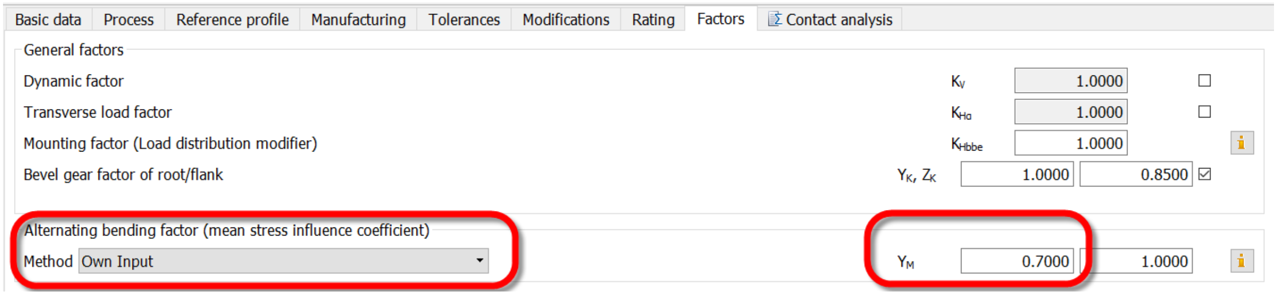


Figure 12. Alternating bending factor for pinion and side gear

2.3.4 Minimum safety factor

The minimum required safety factor differs strongly from the usual numbers such as the recommend value of 1,5 for straight bevel gears (see ISO 10300). As the webbing achieve a huge strengthening effect, the numbers typically are much lower than 1. Safety factors are even possible in the area of 0.3 to 0.5. Therefore, a final test bench run is strongly recommended to verify the capacity of the differential bevel gears.

2.4 Contact analysis

2.4.1 Calculation approach

The contact analysis (loaded tooth contact analysis LTCA) is a meshing simulation under load, whereas for each meshing position a torque equilibrium is found, and the resulting line loads are evaluated. The contact analysis considers the actual tooth form, including modifications such as profile and lengthwise crowning. The evaluations of the contact analysis are the load distribution over the face width, Hertzian pressure etc. but also the transmission error, as the teeth are considered with elastic deformation.

The contact analysis considers the real blank shape of the differential bevel gear. This is a substantial difference to the method of ISO 10300. The flank area of forged bevel gears is reduced considerably by the webbings, what increases the Hertzian pressure. So, the KISSsoft contact analysis provides a fast and accurate method for the rating of the reduced contact area.

As a profile shape of the tooth form, the planar involute is used in KISSsoft. Other profile shape such as octoide and spherical involute, are not implemented in KISSsoft.

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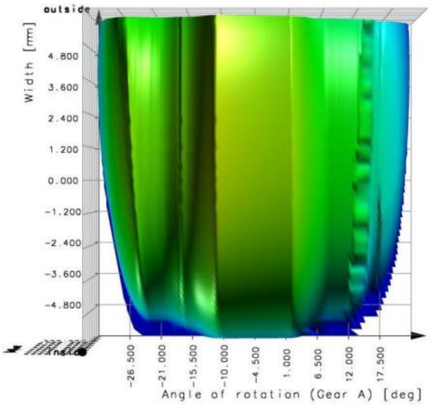
<tr>
<td>
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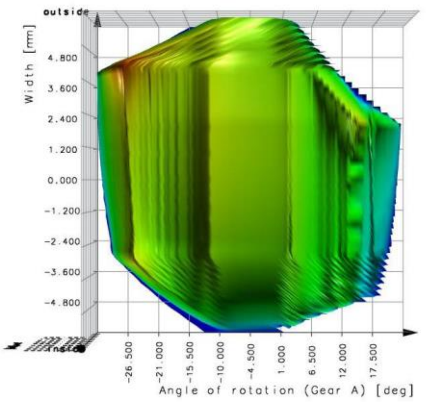
<td>
</td>
</tr>
<tr>
<td>Contact analysis considering the nominal flank shape (no alterations)</td>
<td>Contact analysis considering the modified flank shape (no alterations)</td>
</tr>
</table>

In the KISSsoft contact analysis, the strengthening effect of webbings at the heel and toe side is not considered, as the contact analysis doesn't consider the 3D shape of the blank. This effect has to be considered by a FE calculation. Still, the KISSsoft contact analysis allows an estimation of the root stresses. What allows at least a comparison between the design solutions.

The contact analysis does not provide safety number, as the calculation of permissible stress is missing. This means that the contact analysis is used for qualitative rating, or own numbers for permissible stresses are needed.

2.4.2 Operating data and modifications

The operating data are the same as for the rating with ISO 10300.

The tooth flank modifications such as crowning and profile crowning, are defined in the tab ‘modifications’. By the plus button, the modifications can be added on the selected flank. This allows to rate also the finally applied microgeometry.

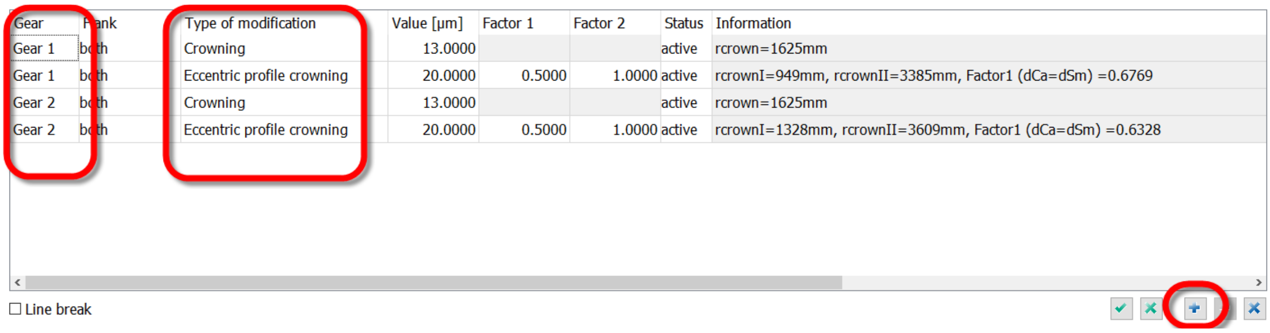


Figure 13. Definition of modification, which are considered in the KISSsoft contact analysis

The cumulated modifications are shown under ‘Graphics – Geometry 3D – modifications – gear 1/2’. Under ‘properties’, the scale for X and Z axis can be set to 50%, which gives the proportions of the graphic similar to the real tooth flank.

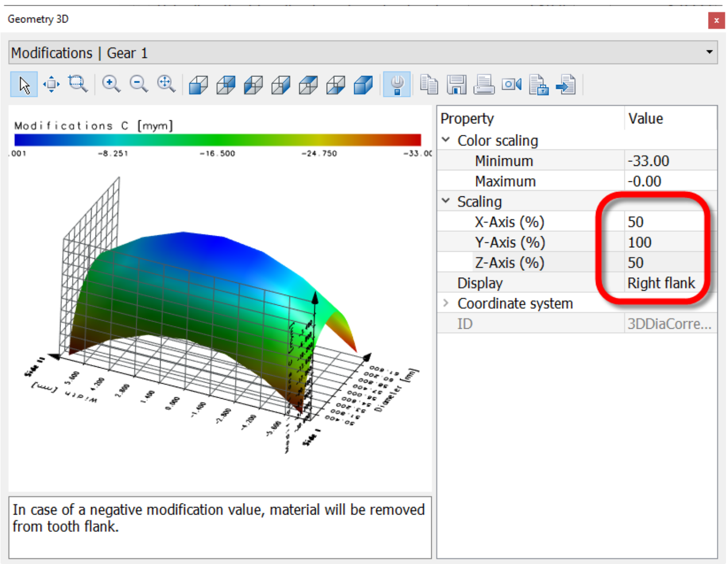


Figure 14. Display of cumulated tooth flank modifications

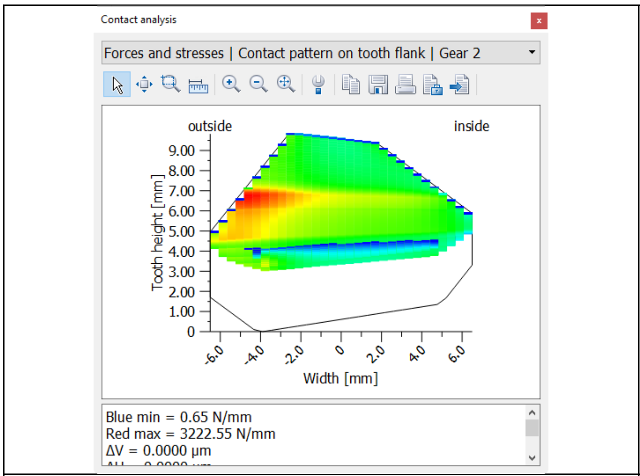
2.4.3 Calculation and results

The calculation of the contact analysis is activated in the tab ‘contact analysis’. By pressing the calculation button, the contact analysis is performed. The results are shown in the result window, the report and the graphics. Typical evaluation values are the peak to peak transmission error and the Hertzian pressure.

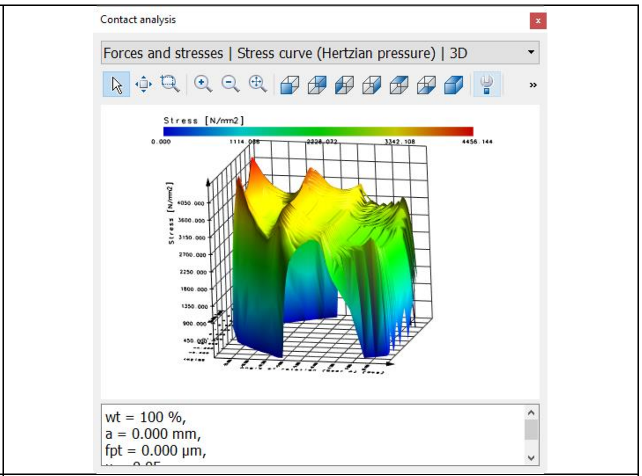
As mentioned above, the results of the contact analysis are qualitatively rating of the contact. Typical graphics are the contact pattern and the Hertzian Pressure distribution.

Note, that the graph for contact pattern represents is based on the line load, where as the graph for stress curve is based on the Hertzian pressure.

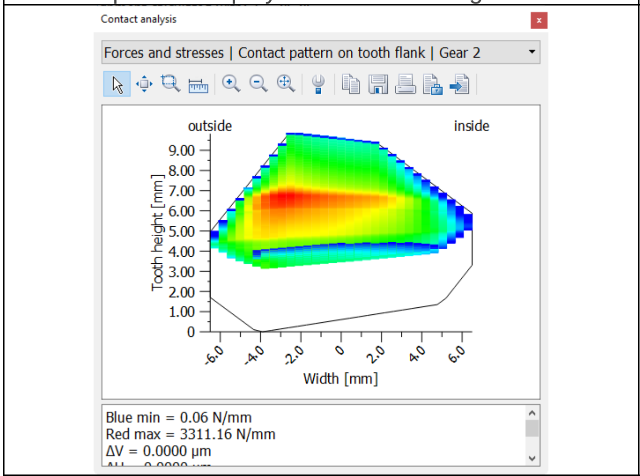
The graphs below show the influence of crowning on the contact pattern and Hertzian pressure. It is obvious, that the stress peaks at begin of mesh are clearly reduced due to the increased crowning.



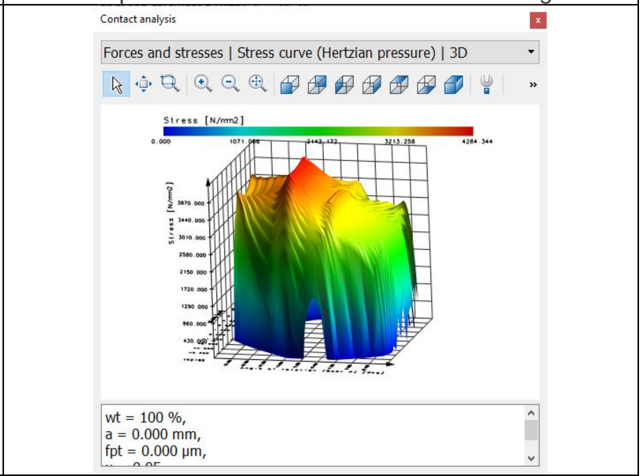
Contact pattern display with low crowning



Hertzian pressure distribution with low crowning



Contact pattern display with high crowning



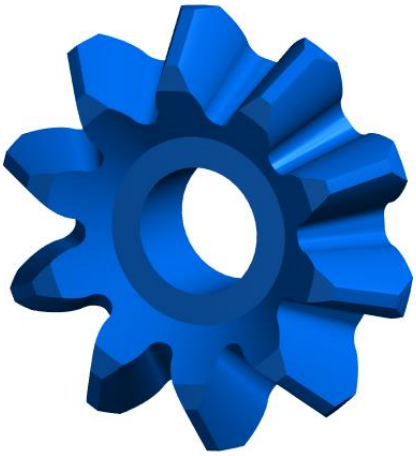
Hertzian pressure distribution with high crowning

Figure 15. Results graphics of the contact analysis

3 FE analysis

A final rating of the differential bevel gears by FE analysis is recommended. The final design of the differential bevel gears are completed in CAD, including also e.g. modifications for forging, radii, etc.

As a basis for modelling in CAD, the KISSsoft 3D models can be used. The export is done in 'Graphics – Geometry 3D – tooth geometry – gear 1,2'. The root alterations are not included in the 3D model, but the tip alterations are. For detailed instruction and individual settings in 3D models, please refer to the KISSsoft instruction 'KISSsoft-anl-068-E-3D Geometry of Spiral Bevel Gear_rev6'.

	
3D model of pinion from KISSsoft	3D model of side gear from KISSsoft