Comparison of Strength Ratings of Plastic Gears by VDI 2736 and JIS B 1759 - In Vision of Building a New International Standard

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Abstract

The demand for strength ratings for plastic gears has been consistently increasing. However, there are no international standards such as ISO, DIN or AGMA but only domestic level or in-house guidelines available. This situation has been a big obstacle in the plastic gear industry. It makes it difficult for engineers to exchange knowledge on design and production which are often done on a global level such as in the electronic and automotive industries.

The only widely accepted strength rating method in western countries had been the German guideline VDI 2545 which was withdrawn in 1996. VDI published a new guideline VDI 2736 in 2014 as the successor to the old guideline. On the other hand, a Japanese standard JIS B 1759 was newly published in 2013 for the calculation of bending load capacity of plastic gears. Both standards are similar in the sense that VDI 2736 is based on DIN 3990 and JIS B 1759 on ISO 6336 which is essentially equivalent to DIN 3990. However, both standards made various adaptations to consider the special characteristics of plastic gears and show differences in several ways.

The main objective of this paper is to clarify the differences of VDI 2736 and JIS B 1759. The comparison will be done only for the bending load capacity since JIS B 1759 does not provide other failure modes such as pitting or wear resistance. Hopefully the paper gives an opportunity to initiate a discussion to establish global consensus on the calculation method for plastic gears and to build a well-accepted international standard in near future.

1. Introduction

The applications of plastic gears are greatly expanding in modern industry as alternatives to metal gears. Plastic gears have various benefits in terms of weight, noise, vibration, lubrication, and in design and production when it is injection molded. At the same time, there are several drawbacks such as lower accuracy, lower strength and higher sensitivity to the operation environment such as temperature and humidity. Both the benefits and the drawbacks mainly stem from its unique material properties. Thus, the strength rating considering the special material properties of the plastics is necessary for the reliable design of plastic gears. Still no international standard is available for the calculation. Every major plastic gear supplier has its own calculation method. This situation is a critical issue for plastic gear industry because it hinders the exchange of product knowledge and information. It is common for the design and the production of plastic gear drives are to be made on a global level. For instance, an automotive company may have multiple suppliers from different countries for the plastic gear drives that are used in their cars. How can the engineers guarantee that all the plastic gear drives have the same level of safety factors and life expectancies if they are designed by different calculation methods?

In western countries, the only widely accepted strength rating method for plastic gears had been the German guideline VDI 2545 [1] which was withdrawn in 1996. After almost 20 years of inactivity, a new guideline VDI 2736 (abbreviated as VDI) was published in 2014 [2] as the successor to the old guideline. On the other hand, a Japanese standard JIS B 1759 (abbreviated as JIS) was published in 2013 [3] for the calculation of bending load capacity of plastic gears. Both standards are quite similar in the sense that VDI is based on DIN 3990-3 (abbreviated as DIN) [4] and JIS on ISO 6336-3 (abbreviated as ISO) [5] which is essentially equivalent to DIN. However, both DIN and ISO apply only for metal gears, and thus several adaptations have been made in VDI and JIS to consider the special characteristics of plastic gear geometry and material. In addition, VDI is based on method C of DIN and JIS on method B of ISO. Consequently, the two standards show differences in several ways.

We will clarify the differences of the standards in detail in the following sections.

2. Comparisons of VDI 2736 and JIS B 1759

2.1 Comparisons of nominal bending stress calculation

The comparisons of nominal bending stress calculation by VDI and JIS are listed in Table 1. First, VDI applies the load influence factors (K factors) while JIS does not. Moriwaki [6] explains that JIS didn't introduce them because dynamic loads would be small and effect of running-in could be large in plastic gears. However, the applications of plastic gears in high speed and high torque conditions are increasing with the development of high performance plastics. It is questionable if we can ignore dynamic loads for those critical applications. At the very least, engineers should be able to make this decision themselves.

Another important difference is the definition of the nominal load. VDI uses nominal tangential load F_t applied on the reference circle while JIS assumes the nominal load F_{wt} is applied on the operating pitch circle.

Table 1 Comparisons of VDI 2736 and JIS B 1759 for bending stress calculation

	VDI 2736	**JIS B 1759**
Bending stress	$\sigma_F = K_F Y_{Fa} Y_{Sa} Y_\varepsilon Y_\beta \dfrac{F_t}{b \cdot m_n}$	$\sigma_F = \dfrac{F_{wt}}{b \cdot m_n} Y_F Y_S Y_\beta Y_f Y_B$
Load factors	$K_F = K_A K_v K_{F\beta} K_{F\alpha}$	Not applied
Nominal tangential load	$F_t = \dfrac{2000T_{1,2}}{d_{1,2}}$	$F_{wt} = \dfrac{2000T_{1,2}}{d_{w1,w2}}$
Tooth form factor	$Y_{Fa} = \dfrac{\frac{6 \cdot h_{Fa}}{m_n} \cdot \cos \alpha_{Fan}}{\left(\frac{s_{Fn}}{m_n}\right)^2 \cdot \cos \alpha_n}$ $Y_{Fa} \approx 2.0$: for internal gear	$Y_F = \dfrac{\frac{6 \cdot h_{Fe}}{m_n} \cdot \cos \alpha_{Fen}}{\left(\frac{s_{Fn}}{m_n}\right)^2 \cdot \cos \alpha_{wt}}$
Stress correction factor	$Y_{Sa} = (1.2 + 0.13L_a) \cdot q_s^{\left(\frac{1}{1.21+2.3/L_a}\right)}$ $L_a = s_{Fn}/h_{Fa}$	$Y_S = (1.2 + 0.13L) \cdot q_s^{\left(\frac{1}{1.21+2.3/L}\right)}$ $L = s_{Fn}/h_{Fe}$
Contact ratio factor	$Y_\varepsilon = 0.25 + \dfrac{0.75}{\varepsilon_\alpha}$	Not applied
Helix angle factor	$Y_\beta = 1 - \varepsilon_\beta \dfrac{\beta}{120^\circ}$	$Y_\beta = 1 - \varepsilon_\beta \dfrac{\beta}{120^\circ}$
Tooth fillet factor	Not applied	If root fillet is based on standard basic rack, $Y_f = 1$. If root fillet is not based on standard basic rack such as radii, $Y_f > 1$. If root is optimized, $Y_f < 1$.
Rim thickness factor	Not applied	External gears: If $s_R/h_t \geq 1.4$, $Y_B = 1$. If $0.4 \leq s_R/h_t < 1.4$, $Y_B = 0.276\, ln(52.9\, h_t/s_R)$. Internal gears: If $s_R/h_t \geq 1.5$, $Y_B = 1$. If $0.4 \leq s_R/h_t < 1.5$, $Y_B = 0.759\, ln(5.61\, h_t/s_R)$.
Deep tooth factor	Not applied	Not applied

JIS explains that this is because that the load capacity of a gear should be determined in terms of the strength of a gear pair, not a single gear. According to this change, JIS also modified the tooth form factor Y_F to use the transverse pressure angle at the pitch circle α_{wt} instead of the normal pressure angle α_n. JIS explains that this change is first made on the pressure angle from normal to transverse after validating the formula in ISO, and then from reference to operating according to the usage of F_{wt}. However, as the operating pitch diameter is defined as $d_{wt} = d\frac{\cos\alpha_t}{\cos\alpha_{wt}}$, and $F_t/\cos\alpha_t$ and $F_{wt}/\cos\alpha_{wt}$ are the same as shown in Table 2. Thus, the changes to use F_{wt} and α_{wt} is meaningless.

Table 2 Differences by using operating pitch circle for nominal load and pressure angle

	α_t	α_{wt}	F_t	F_{wt}	$\frac{F_t}{\cos\alpha_n}$	$\frac{F_t}{\cos\alpha_t}$	$\frac{F_{wt}}{\cos\alpha_{wt}}$
$T_1 = 10\text{Nm}, m_n = 1,$ $z_1 = 17, z_2 = 50, \alpha_n = 20°,$ $\beta = 0°, x_1 = 1.0, x_2 = 0.0$	20.0	23.831	1176.5	1145.2	1252.01	1252.01	1252.01
$T_1 = 50\text{Nm}, m_n = 3,$ $z_1 = 35, z_2 = 50, \alpha_n = 20°,$ $\beta = 30°, x_1 = 1.0, x_2 = 0.0$	22.796	25.261	824.8	809.1	877.73	894.68	894.68

There is another difference in the tooth form factor Y_F and in the stress correction factor Y_S. In calculating both factors, VDI assumes that the load is applied at the tooth tip when calculating the geometry factors while JIS takes the load applied at the highest point of single tooth contact. The VDI's approach follows the method C in DIN and gives a more conservative result (lower safety) to consider lower quality and high dimensional variation of plastic gears. However, this approach is questionable as new materials with better mechanical properties have been developed in recent years and the advances in design and manufacturing technologies has shown that high quality plastic gears can be achieved. As a compromise, it is preferable to allow the engineer to choose the load application point. The tooth form factor Y_F for internal gears are approximated as 2 in VDI while JIS follows so-called 60° tangent method per ISO. Clearly the approach of VDI might be regarded as too simplified.

Both VDI and JIS use the helix angle factor as ISO to convert the tooth root stress of a virtual spur gear to that of the corresponding helical gear.

VDI uses the contact ratio factor Y_ε according to method C in DIN. The factor is used to convert the stress calculated by the tooth form factor and the stress correction factor for application of load at the tooth tip to a value approximating the condition where determinant position of load is at the outer point of single pair tooth contact. JIS is based on ISO method B and there is no need to include the contact ratio factor.

JIS newly introduced the tooth fillet factor Y_f that was not included both in VDI and ISO. This is to consider the change in root stress if the root fillet is not defined by the standard basic rack. The introduction of the factor might be regarded as the proper approach since injection molded plastic gears can have various fillet shape. JIS defines $Y_f > 1$ if the root fillet is not based on the standard basic rack such as radii. If the root fillet is optimized, then $Y_f < 1$. However, the calculation formulas are not yet given. Only empirical formulas shown in the annex based on FEM for the cases of arc shaped fillet giving $Y_f > 1$ are available. It is common practice for plastic gears to optimize root fillet shape such as elliptical curves to have bigger radius than the fillet cut by the basic rack. It should be possible to provide the general calculation formula considering arbitrary fillet shape.

JIS applies the rim thickness factor Y_B by using the modified formula from ISO as shown in Table 1. Moriwaki [6] explains that the modification has been made by using the results from operating tests and FEM to consider the lower stiffness of plastic gears relative to the metal gears. Figure 1 shows graphical comparison of the factor according to the backup ratio for external and internal gears. As the rim thickness factor for internal gear is defined as the ratio of normal module in ISO, we converted the factor as the ratio of the tooth height assuming $h_t = 2.25m_n$ of the standard basic rack. The figure shows the effect of the rim thickness is considerably smaller in both external and internal gears. VDI doesn't apply the rim thickness factor, the same as DIN method C.

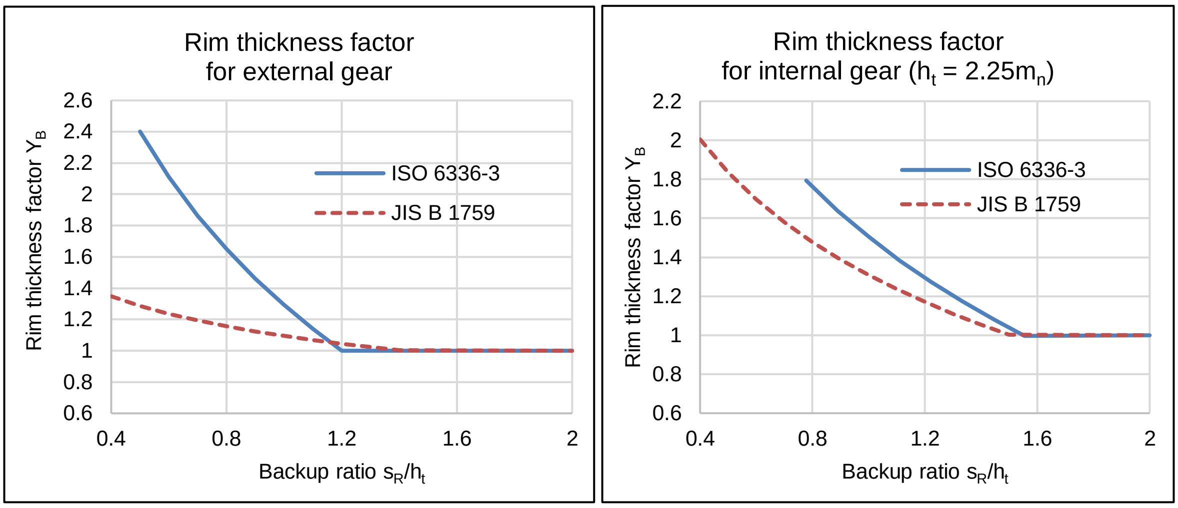

Figure 1 Comparison of rim thickness factors by ISO 6336-3 and JIS B 1759

Neither VDI nor JIS apply the deep tooth factor which is used in ISO and DIN. This is because the deep tooth factor is only meaningful for high precision gears with accuracy grade equal or less than 4 which is generally difficult to achieve in plastic gears.

2.2 Comparisons of permissible bending stress calculation

The comparisons of the permissible bending stress calculation are listed in Table 3.

Table 3 Comparisons of VDI 2736 and JIS B 1759 for permissible bending stress calculation

	VDI 2736	**JIS B 1759**
Permissible bending stress	$\sigma_{FG} = \sigma_{Flim} Y_{NT} Y_{ST} = \sigma_{FlimN} Y_{ST}$ $\sigma_{FP} = \dfrac{\sigma_{FG}}{S_{Fmin}}$	$\sigma_{FP} = \sigma_{Flim} Y_{NT} Y_{\Theta} Y_{\Delta\Theta} Y_L Y_M$
Allowable bending stress	σ_{Flim} defined as stress level with 10% failure probability	σ_{Flim} defined as stress level with 1% failure probability for 10^6 load cycles
Service life factor	Y_{NT} combined into σ_{FlimN}	$Y_{NT} = \dfrac{\sigma_F}{\sigma_{Flim}}$
Stress correction factor	$Y_{ST} = 2.0$	Not applied
Temperature factor	Not applied	$Y_{\Theta} = \dfrac{\sigma_{F\,lim(\text{at } \Theta = XX°\text{C})}}{\sigma_{Flim}}$ $Y_{\Theta} = 1$ for standard test condition.
Temperature rise factor	Not applied	$Y_{\Theta} = \dfrac{\sigma_{Flim\Theta}}{\sigma_{Flim0}}$ $Y_{\Delta\Theta} = 1$ for standard test condition. If $m_n < 1$, $b <$ 8mm, n < 1000 rpm, $Y_{\Delta\Theta} > 1$. If $m_n \geq 1$, $b \geq$ 8mm, $n \geq$ 1000 rpm, $Y_{\Delta\Theta} < 1$.
Lubrication factor	Not applied	$Y_L = 1$ for standard test condition without lubrication. Otherwise, $Y_L > 1$.
Material factor	Not applied	If mating gear is steel, $Y_M = 1$. If mating gear is plastic, $Y_M < 1$. If σ_{Flim} is taken from the plastic-plastic gear pair test and the mating gear is steel, $Y_M > 1$.

Both VDI and JIS defines the calculation method for permissible bending stress σ_{FG} and σ_{FP} based on the allowable bending stress σ_{Flim} measured from gear test rig. Note that VDI assumes the failure probability of 10% for the assessment of the measured data for the allowable stress while JIS assumes that of 1%. JIS does not provide any data for allowable bending stress while VDI provides for four different materials (POM, PA 66, PET, PE).

The service life factor Y_{NT} is applied to the data for σ_{Flim} to obtain the allowable bending stress at the required number of load cycles in the limited life region. Neither VDI nor JIS provide a general formula for the factor Y_{NT}. Instead, VDI provides the data and respective equations of σ_{FlimN} directly including the number of load cycles for PA 66 and POM considering temperature given in Equations (1) and (2).

$$\sigma_{FlimN} = 30 - 0.22 \cdot \vartheta + (4600 - 900 \cdot \vartheta^{0.3}) \cdot N_L^{-1/3};\ N_L \in [10^5; 10^8] \text{ for PA 66} \quad (1)$$

$$\sigma_{FlimN} = 26 - 0.0025 \cdot \vartheta^2 + 400 \cdot N_L^{-0.2};\ N_L \in [10^5; 10^8] \text{ for POM} \quad (2)$$

On the other hand, JIS calculates the permissible bending stress based on the allowable bending stress σ_{Flim} with the temperature factor Y_Θ, the temperature rise factor $Y_{\Delta\Theta}$, the lubrication factor Y_L, and the material factor Y_M as shown in Table 3. JIS does not provide any data for the allowable bending stress σ_{Flim} and the calculation formulas for the factors except general comments on the decision criteria. Instead, it provides calculation examples for the factors based on test results for POM test gears meshing with steel gears in its Annex. For instance, the Annex shows the formula for the allowable bending stress, the service life factor and the temperature factor as shown in Equations (3) - (5). It also shows specific values for the temperature rise factor, the lubrication factor and the material factor but more extensive work shall be made to obtain an estimation formula.

$$\sigma_{Flim} = 376 \cdot N_L^{-0.112} \quad (3)$$

$$Y_{NT} = 4.70 \cdot N_L^{-0.112} \quad (4)$$

$$Y_\Theta = (-3.12 \cdot 10^{-3}) \cdot \Theta + 1.07 \quad (5)$$

Essentially the calculation of the permissible bending stress in both standards has the same concept that the stress is represented as a function of temperature, torque and load cycles, but JIS might be said to have more proper structure for further investigation of each operating parameters.

VDI applies the stress correction factor Y_{ST} from the reference test gears to obtain the permissible bending stress σ_{FG} while JIS doesn’t apply the factor. VDI sets $Y_{ST} = 2.0$, the same as DIN and ISO.

The setup and test condition for the gear test rig is shown in Table 4. Both standards allow different types of test rigs but prefer mechanically non-closed loop type (power absorption type) test rigs. JIS defines the standard test condition more specifically. Considering the large number of plastic materials and cost for the test, it is almost impossible to include complete set of data into the standards. However, the formal definition of the test procedure makes it inevitable to gain a reliable material database.

For the test gears, VDI shows three different types (Size 1, Size 2, Size 3) based on the work from respective sources while JIS specifies only one dimension. Table 5 shows the comparison of the dimensions of the test gears from VDI and JIS. For VDI, only the type Size

1 is shown since the normal module of it is the same as that from JIS ($m_n = 1$). The biggest difference is the number of teeth. The test gear in VDI has the number of teeth of 17 while JIS specifies relatively large number of teeth ($z_1 = 50$). It is difficult to assess which test gear is more suitable for the test, but at least the test gear in VDI has a benefit to reduce the testing time. Note that the test gear in VDI has positive profile shift coefficient ($x_1 = 0.259$), presumably to prevent undercut. The allowable stress data of plastic gear materials is most important in strength rating. The standardization of the test gears together with the test setup will surely accelerate the process to obtain reliable data.

Table 4 Comparison of test rig setup and standard test condition

	VDI 2736	**JIS B 1759**
Test rig type	Mechanically non-closed loop type (Power absorption type) Mechanically closed loop type (Power circulation type)	
Rotation speed	Not specified	1000 rpm (meshing with steel gear) 500 rpm (meshing with plastic gear)
Ambient temperature	Not specified	23±2°C
Relative humidity	Not specified	50±5%
Lubrication	Not specified	Dry running

Table 5 Comparison of test gears

Quantity	**Symbol**	**Unit**	**VDI 2736, Size 1**	**JIS B 1759**
Center distance	a	mm	28	–
Normal module	m_n	mm	1	1
Number of teeth	z_1; z_2	–	17; 39	50; ≥ 50
Face width	b_1; b_2	mm	8; 6	8; ≥ 8
Tip diameter	d_{a1max}; d_{a1min}	mm	19.40; 19.35	–
	d_{a2max}; d_{a2min}	mm	40.40; 40.30	–
Root diameter	d_{f1max}; d_{f1min}	mm	14.902; 14.610	–
	d_{f2max}; d_{f2min}	mm	35.866; 35.691	–
Tip rounding radius	r_{k1}; r_{k2}	mm	0.0; 0.08	–
Profile shift coefficient	x_1; x_2	–	0.259; -0.259	–
Pressure angle	α_n	°	20	20
Helix angle	β	°	0	0
Reference profile factors	h^*_{aP1}; h^*_{aP2}	–	0.94; 0.96	1.0; –
	h^*_{fP1}; h^*_{fP2}	–	1.25; 1.25	1.25; –
	ρ^*_{fP1}; ρ^*_{fP2}	–	0.25; 0.25	0.38; –

Base tangent length (Number of teeth for measurement)	W_{k1max}; W_{k1min}	mm	7.756; 7.656 (3)	–
	W_{k2max}; W_{k2min}	mm	10.662; 10.602 (4)	–
Manufacturing method			Injection molded	Injection molded
Tooth quality	test gear		DIN 58405, 10	–
	mating gear		Steel: DIN 3961, 6	Steel: JIS B 1702, 5

3. Conclusions

This paper clarified the differences between VDI 2736 and JIS B 1759 for the bending load capacity of plastic gears. Both the standards have its own merits and it is not easy to state which standard is superior to the other. Based on the comparison in this paper, however, the authors are hoping to initiate a discussion to build a global consensus on the strength rating method for plastic gears. It cannot be emphasized enough that a well-established international standard is most important for the rapid evolution of plastic gear technology.

Nomenclature

Symbol	Description	Unit
b	Face width	mm
$d_{1,2}$	Reference circle of pinion and gear, respectively	mm
$d_{w1,w2}$	Operating pitch circle of pinion and gear, respectively	mm
h_{Fa}	Bending moment arm relevant to load application at the tooth tip	mm
h_{Fe}	Bending moment arm relevant to load application at the outer point of single pair tooth contact	mm
h_t	Tooth height	mm
F_t	Nominal tangential load at reference circle	N
F_{wt}	Nominal tangential load at operating pitch circle	N
K_A	Application factor	–
K_v	Dynamic factor	–
$K_{F\beta}$	Face load factor	–
$K_{F\alpha}$	Transverse load factor	–
K_F	Tooth root load factor	–
m_n	Normal module	mm
q_s	Notch parameter	–
S_{Fmin}	Required minimum safety factor	–
s_{Fn}	Tooth root chord at the critical section	mm
s_R	Rim thickness	Mm
$T_{1,2}$	Nominal torque of pinion and gear, respectively	N·m

Y_B	Rim thickness factor	–
Y_F, Y_{Fa}	Tooth form factor	–
Y_f	Tooth fillet factor	–
Y_L	Lubrication factor	–
Y_M	Material factor	–
Y_{NT}	Life factor	–
Y_S, Y_{Sa}	Stress correction factor	–
Y_{ST}	Stress correction factor relevant to the reference test gear	–
Y_β	Helix angle factor	–
Y_ε	Contact ratio factor	–
Y_Θ	Temperature factor	–
$Y_{\Delta\Theta}$	Temperature rise factor	–
α_{Fan}	Pressure angle at the tooth tip	°
α_{Fen}	Pressure angle at the outer point of single pair tooth contact	°
α_n	Normal pressure angle	°
α_{wt}	Transverse pressure angle at the operating pitch cylinder	°
β	Helix angle at reference circle	–
ε_α	Transverse contact ratio	–
ε_β	Overlap ratio	–
ϑ, Θ	Ambient temperature	°C
σ_F	Tooth root stress	N/mm^2
σ_{Flim}	Fatigue strength (nominal root stress)	N/mm^2
σ_{FlimN}	Fatigue strength of the required number of load cycles	N/mm^2
σ_{FP}	Permissible tooth root stress	N/mm^2

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