

# Evaluation of the NVH Characteristics of Gear Drives with Plastic Gears by the Forced Response Analysis

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## **Abstract:**

This paper presents some guidelines to properly adjust the gear meshing damping for achieving better NVH characteristics of a powertrain system. This goal can be achieved by choosing the gear body material with higher damping properties within the required torque ratio specifications. For this purpose, a two-stage gearbox model is considered in some different schemes. Two variants of the model with different gear material, one with plastic gears and one with steel gears, are considered which result in the same order of tooth root and flank safety factors. The forced response analysis of the models is carried out in KISSsoft and the bearing reaction forces are calculated to evaluate which model results in lower noise emission from the housing. For characterization of the emitted noise of the models, the exciting bearing forces of the forced response are then imported to RecurDyn and applied to the housing of the powertrain model. In this way, one can explicitly investigate the effect of the gear material damping on the level of noise emitted directly from the housing surface. The results confirm that the gearbox with plastic gears compared to the steel gears can lead to a significant reduction of the noise emission by damping out the structure-borne noise transferred from the gear meshes to the bearings.

## **1. Introduction:**

In powertrain systems, all deformable elements can dissipate energy when subjected to dynamic deformations. The internal damping of these elements including bearings and supports, shafts and gears represents the energy dissipation. Consequently, damping plays an important role in the design of structures to minimize noise, structural instability, and fatigue failure of components. The gear meshing damping is known as an important determinative factor in dynamic response of the system under some specific running conditions. With the viscous meshing damping, the effect of lubricant film as well as the internal damping of the gear teeth in dissipating the kinetic energy into frictional heat and constraining the motion amplitudes can be modeled. The accuracy of the natural frequencies and resonance frequencies are influenced slightly by the actual value of the damping ratio. However, its impact on the resonance peak amplitudes is significant. In addition, the nonlinear behavior induced

by backlash and parametric resonances associated with the time-varying gear meshing stiffness are very sensitive to the damping value of the gear mesh.

Mainly three sources are known to cause damping in gear meshes: internal hysteresis of the teeth (internal damping), oil squeeze damping, and surrounding element contribution [1]. In many applications, a user-defined constant viscous damping ratio is employed to represent the damping element. However, a non-constant meshing viscous damping can also be used. This choice of damping is considered an improvement of the constant damping factor and can result in more realistic dynamic responses. Accordingly, substantive research on this subject has been carried out to extend the constant viscous damping model to include time-varying damping. In this context, a non-linear damping model with backlash and under no lubrication condition for vibration characteristics of a light-loaded spur gear pair was presented in [2]. It was observed that the induced vibrations of the model with non-linear damping are much greater than the model with linear damping under some conditions. Amabili and Rivola [3] presented a single DOF model of a pair of low contact ratio spur gears with a time-varying meshing viscous damping with the inclusion of gear errors. The damping was assumed to be proportional to the meshing stiffness of the tooth pairs. Linear and nonlinear damping in the dynamics of gear mesh of the parametric systems with impacts was also presented in [4]. The aim of their contribution was to analyze the damping properties considering both the material of the gear mechanism in the mesh and the lubricating oil film in tooth space at tooth profile contact. Li and Kahraman [5] proposed a viscous damping formulation for a spur gear pair based on a transient gear elastohydrodynamic lubrication model and a torsional dynamic model. A simplified expression for the damping ratio of the gear pair was derived along the line of action direction. The viscous damping was considered to model the instantaneous tribological behavior of the tooth contacts. The shear stress distributions along the tooth surfaces were incorporated with the dynamic model to formulate the periodic gear mesh viscous damping. In addition, the effect of various contact conditions including speed, torque and lubricant temperature on the resultant gear meshing damping value were demonstrated. They observed that the mean and alternating values of the gear meshing damping increase with increased torque while decrease with increased rotational speed and lubricant temperature. In addition, the damping value reduces with increased speed, primarily due to the increased film thickness. Guilbault et al. [6] presented a method for calculating nonlinear damping in cylindrical gear systems. The nonlinear damping ratio calculated at different meshing frequencies and torque amplitudes presented average values between 5.3 percent and 8 percent. Later, an analytical method for calculating the variation of the damping ratio in gear teeth contacts under lightly loaded hydrodynamic conditions was presented to reflect the

lubricant effects [7]. They characterized analytically damping and stiffness in lightly loaded gears for vibro-impact regimes at different speeds and lubricant temperatures. In addition, the various damping mechanisms in elastohydrodynamic lubrication contacts were investigated in [8] with the objective of deriving damping models representative of the lubricant contributions which can readily be used in the gear dynamic simulations. Yousfi et al. [9] developed an advanced formulation for viscous damping in the time frequency domain. They discussed the capability of the continuous wavelet transform (CWT) method to select optimum frequencies of viscous damping formulation considering the classical Rayleigh, modal and the proposed full model modal damping. Studies proved that the damping ratio depends on the operating conditions such as load. More recently, Yousfi et al. [10] estimated the damping model of a spur gear pair system including time-varying loading conditions.

Without damping, undesirable induced vibrations create noise, increase dynamic loads, and potentially damage the gear teeth and bearings. Therefore, investigation of noise, vibration, and harshness (NVH) in the presence of viscous gear meshing damping is demanded in dynamic models with plastic gears. It is well known that the noise emission level of a gearbox is directly related to the excitation behavior of the powertrain. The dynamic interaction between the meshing teeth and the structural behavior of the drivetrain characterizes the noise and vibration behavior. This subject has recently received some attention from researchers. In this context, a comprehensive procedure was presented in [11] to characterize vibration and noise radiation of gear transmission by calculating steady dynamic response and noise radiation. The effect of the excitation of second harmonic of meshing frequency, load torque and rotation speed on the gearbox vibration and noise radiation was also investigated. A constant damping model proportional to the meshing stiffness was used by Bozca [12] to reduce the gear rattle noise in an automotive transmission gearbox. For this purpose, a transmission error model was used to optimize some gearbox geometric design parameters. The work of Martin et al. [13] investigated the effect of gearbox parameters on the noise of transmission. The impact of torque amount, imbalance position, backlash size, and torsion stiffness of each part was mainly investigated. Their method was able to determine the impact of each modification, thus it can be useful at the design phase and during the measurement or investigation of a real problem. O'Rourke, and Grander [14] investigated the effect of material selection to reduce gear noise. Later, the damping characteristics of unreinforced, glass and carbon fiber reinforced nylon 6/6 spur gears were studied in [15]. More recently, Sharma et al. [16] investigated the effect of the damping properties of gear material and noise level in spur gears for light load application in automotive vehicles. Their results indicated that glass fiber reinforced plastic spur gears have

better NVH characteristics than the metal gears in light load powertrains due to their lower noise emission.

This paper presents some guidelines to properly adjust the gear meshing damping for achieving better NVH characteristics of a powertrain system. In this regard, a dynamic calculation process, called forced response analysis tool, has been developed in KISSsoft. To achieve this goal, gear body material with higher damping properties within the required torque ratio specifications can be chosen. For this purpose, a two-stage gearbox model is considered in some different schemes. The forced response analysis of the models is carried out and the exciting bearing forces are calculated to evaluate which model results in lower noise emission from the housing. Based on the load excitation imposed from gears, the forced response analysis is conducted and at the bearing positions, the transient loads are evaluated. In case of the NVH evaluation, furthermore the housing is excited by the transient bearing forces, and consequently, the noise emitted from the housing is evaluated.

## 2. Model Setup in KISSsoft

The system analyzed in the following is a two-stage gearbox which can be used as a speed reducer while increasing the torque, as shown in Figure 1. Two variants of the model with different gear materials, one with plastic gears and one with steel gears, are considered. Both variants have the same input parameters (load, speed, number of cycles) and are designed so that to have similar root and flank safeties. Therefore, the plastic gears are larger and have bigger diameters. The plastic gears strength is calculated considering VDI 2736, while the steel gears are calculated using ISO 6336. Both variants have housing from the same material. The analysis is carried out for the constant input torque of 0.39 Nm applied to the input shaft, which is considered here as the reference boundary with a nominal speed of 2742 rpm. Tables 1 and 2 specify the gear mesh data of the gearbox for both variants.

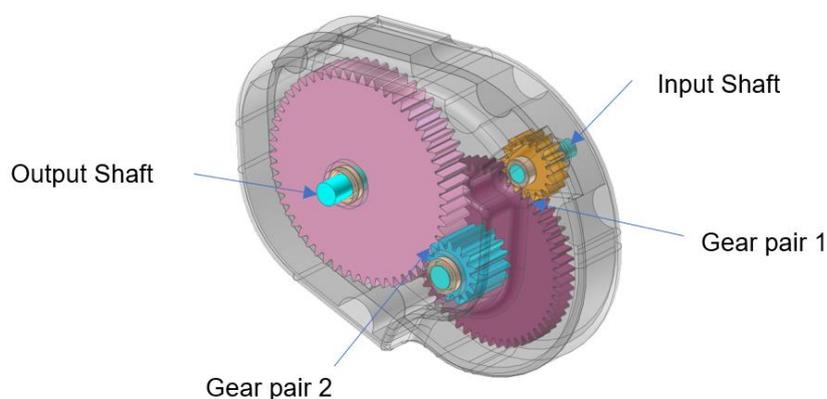


Figure 1: A two-stage gearbox model layout

Table 1: Gear mesh data of the gearbox with plastic gears

Gear parameters		Gear pair 1		Gear pair 2	
		Gear 1	Gear 2	Gear 3	Gear 4
Teeth number	$z$ [-]	17	56	15	56
Face width	$b$ [mm]	8	7	17	12
Center distance	$a$ [mm]	40.15		45.48	
Normal module	$m_n$ [mm]	1.1		1.25	
Pressure angle	$\alpha_n$ [°]	20		20	
Profile shift	$x^*$ [-]	0.4337	-0.4337	0.3119	0.6487
Excitation order	GMF	17		4.55	
Gear profile	$h_{fp}^*/\rho_{fp}^*/h_{ap}^*$	1.5/0.2/1.1		1.35/0.2/1.1	
Contact ratios	$\varepsilon_\alpha/\varepsilon_\beta/\varepsilon_\gamma$	1.68/0/1.68		1.53/0/1.53	
Safety factors	$S_F/S_H$	4.044/1.510	3.990/1.825	2.550/1.420	2.570/1.730

Table 2: Gear mesh data of the gearbox with steel gears

Gear parameters		Gear pair 1		Gear pair 2	
		Gear 1	Gear 2	Gear 3	Gear 4
Teeth number	$z$ [-]	17	56	15	56
Face width	$b$ [mm]	8	7	13	12
Center distance	$a$ [mm]	23.737		28.975	
Normal module	$m_n$ [mm]	0.65		0.8	
Pressure angle	$\alpha_n$ [°]	20		20	
Profile shift	$x^*$ [-]	0.4357	-0.4172	0.4204	0.3503
Excitation order	GMF	17		4.55	
Gear profile	$h_{fp}^*/\rho_{fp}^*/h_{ap}^*$	1.5/0.2/1.1		1.35/0.2/1.1	
Contact ratios	$\varepsilon_\alpha/\varepsilon_\beta/\varepsilon_\gamma$	1.676/0/1.676		1.540/0/1.540	
Safety factors	$S_F/S_H$	4.609/0.794	3.860/0.900	2.865/0.702	3.101/0.851

Table 3 shows the bearings specifications of both models, as both gearbox variants have the same bearings. In this Table, following parameters are specified:  $D_i$ : inner diameter,  $D_o$ : outer diameter,  $B$ : width,  $C$ : dynamic load rating,  $C_o$ : static load rating,  $P_{do}$ : nominal diametral clearance, and  $P_{ao}$ : nominal axial clearance. The damping of bearings is neglected here to mainly focus on the effect of gear meshing damping to clearly illustrate the effect of gear material difference.

Table 3: Ball bearings data of the gearbox model

Bearing	$D_i$ (mm)	$D_o$ (mm)	$B$ (mm)	$C$ (kN)	$C_o$ (kN)	$P_{do}$ ( $\mu\text{m}$ )	$P_{ao}$ ( $\mu\text{m}$ )
B1, B2 (Input shaft)	6.35	9.525	3.175	0.212	0.088	7.50	37.997
B3, B4 (Intermediate shaft)	8.0	12.0	2.50	0.312	0.140	7.50	41.758
B5, B6 (Output shaft)	8.0	12.0	2.50	0.312	0.140	7.50	41.758

### 3. Forced response analysis

The forced response analysis module in KISSsoft [17] helps the analysts and engineers to perform the dynamic analysis of the powertrain systems quickly and efficiently. Based on the static transmission error of the gears, shaft imbalances, etc., the transient bearing loads are calculated considering the inertias and masses. Considering flexible shafts, the calculation is done in the frequency domain, which allows to simulate the powertrain system for different running speeds. Within this tool, a comprehensive list of different settings and options are provided which enables the user to precisely investigate the vibration characteristics of the system. With the current implementation, the forced response analysis of powertrains with helical and bevel gears mounted on normal and coaxial shafts with switchable or coupling connection elements can be accomplished. This tool is very efficient when the overloading effects on gears and quick estimation of the emitted noise are required. A major advantage of this dynamic calculation is to check the drivetrains for critical frequencies, which can create both noise and overload excitations.

As a first step, the forced response analysis of both models without gear meshing damping is carried out for a speed range of the input shaft from 100 rpm to 6000 rpm. This speed range is selected to ensure that the excitation frequencies are in subcritical areas, which allows a more precise comparison between the two models. The hysteresis material damping of the flexible shafts in the torsional, axial, and bending directions is  $10^{(-5)}$  seconds.

Depicted in Fig. (2) is the dynamic factor  $K_v$  [18] without gear meshing damping for both models. The dynamic factor, which is typically defined as the ratio of the maximum dynamic excitation loading between the meshing gears to the static contact force, characterizes the system behavior under dynamic loading at different shaft speeds and reveals the margins of the operational speeds for which the powertrain system can be significantly excited. As it can be seen from Fig. 2, the increase of the dynamic meshing contact forces compared to the static contact forces in the model with plastic gears are about 100 times higher than the other model when no gear meshing damping is considered.

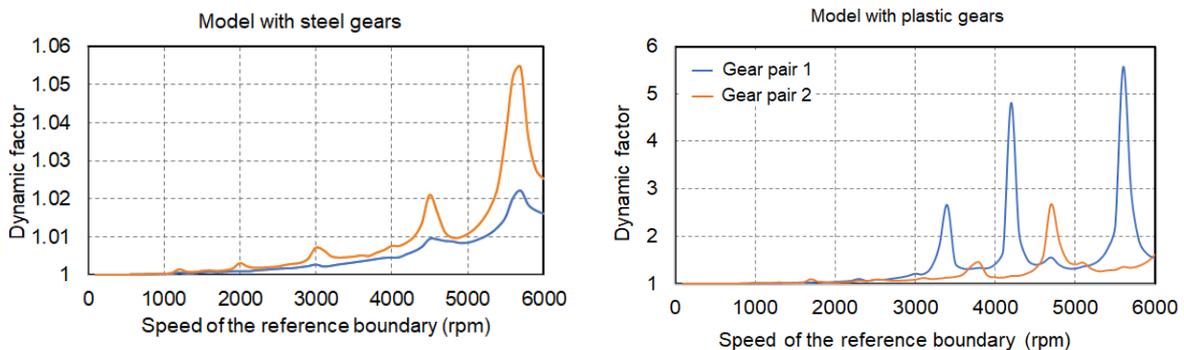


Figure 2: Dynamic factor without gear meshing damping,  $d=0$  Ns/m

According to Figure 3, the main reason behind this difference is the high transmission error and low meshing stiffness of the plastic gears compared to the steel gears. The difference of the transmission error between these models implies that the dynamic meshing forces of the plastic gears without damping consideration, can be significantly higher than the steel gears.

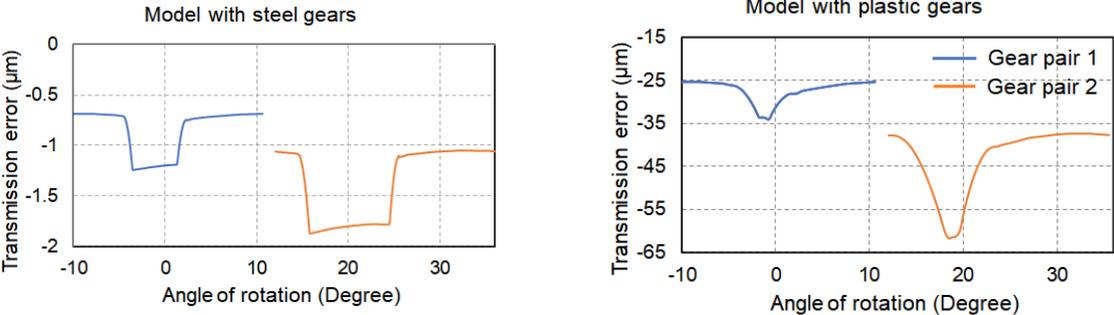


Figure 3: Transmission error of the gear pairs

In the next step, the dynamic factor is calculated for different meshing damping factors 1000 Ns/m, 5000 Ns/m and 10000 Ns/m. Figures 4 and 5 depict clearly that the effect of damping on the dynamic meshing contact forces is not remarkable for the gearbox with steel gears. On the other hand, increasing the damping considerably decreases the dynamic factors of the gearbox model with plastic gears. It is interesting to notice that the effect of damping at lower values is dominant, which means that the reduction of the dynamic factor from  $d=0$  Ns/m to 1000 Ns/m is higher than reduction from 1000 Ns/m to 5000 Ns/m. The same conclusion can be drawn by comparing the dynamic factor reduction from successive damping increase. It is however noticeable to mention that these values of the damping are considered to qualitatively assess the effect of energy loss at meshing teeth on the maximum dynamic meshing contact forces for both models. As the main conclusion from the above analysis, it reveals that for plastic gears, an analysis without damping is absolutely useless. This implies that the damping should not be neglected even when using an approximate damping is possible, otherwise it would lead to the unrealistic response of the system.

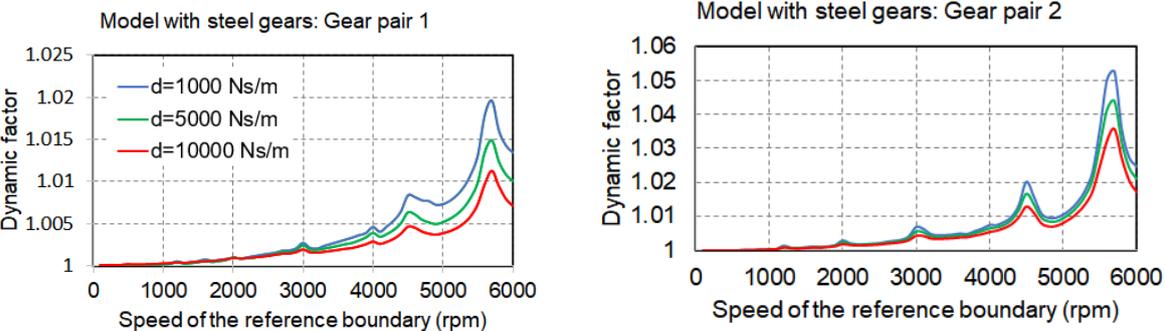


Figure 4: Dynamic factor of the steel gears model with different gear meshing damping

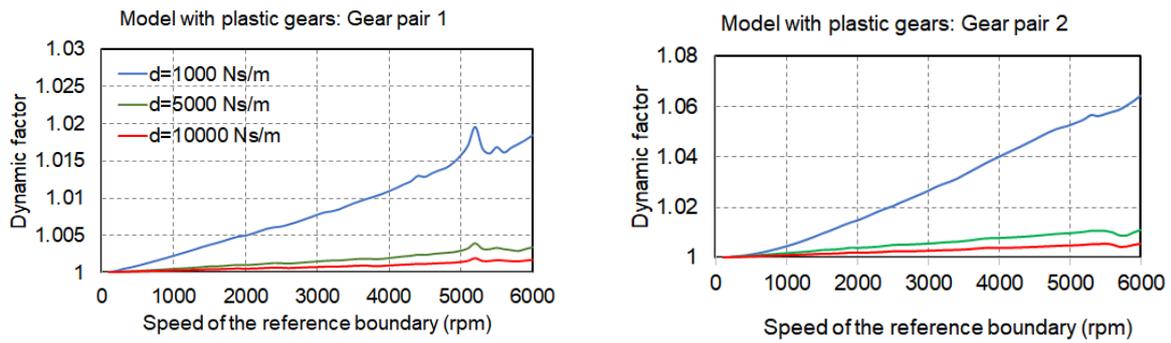


Figure 5: Dynamic factor of the plastic gears model with different gear meshing damping

On the other hand, for steel gears, it is not crucial as plastic gears to use damping, since the results are not much affected when compared to the plastic gears.

Variation of the double amplitudes (total difference between the maximum and minimum values) of the dynamic meshing contact forces of both models with respect to the speed of the reference boundary is shown in Figs. 6 to 8 without and with gear meshing damping. The same behavior as the dynamic factor can be observed. The importance of checking the double amplitudes of the forces is relied on this fact that the NVH characteristics of a gear transmission system is directly related to the fluctuations of the meshing contact forces. This implies that by reducing the amplitudes of the dynamic meshing contact forces, less noise is expected to be emitted from the housing.

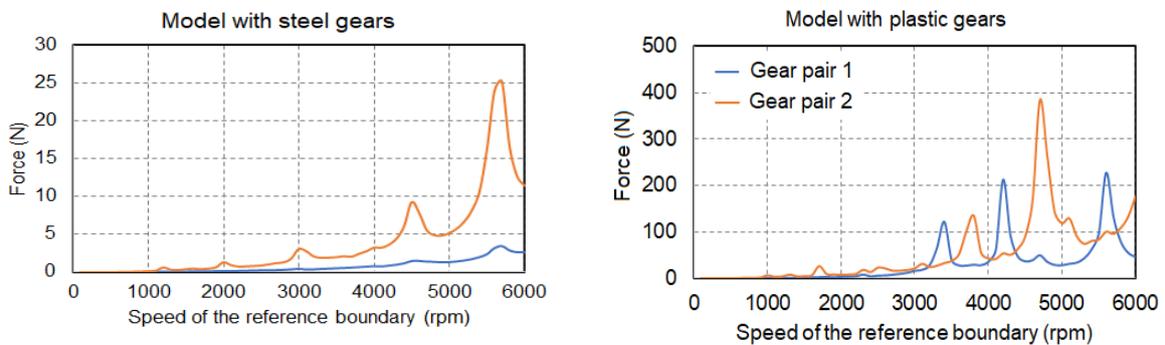


Figure 6: Double amplitudes of the dynamic meshing contact forces for  $d=0$  Ns/m

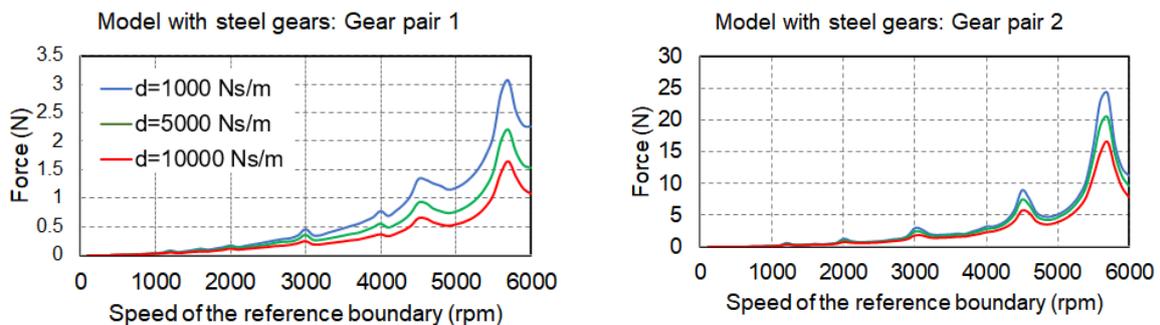


Figure 7: Double amplitudes of the dynamic meshing contact force of the steel gears model with different gear meshing damping

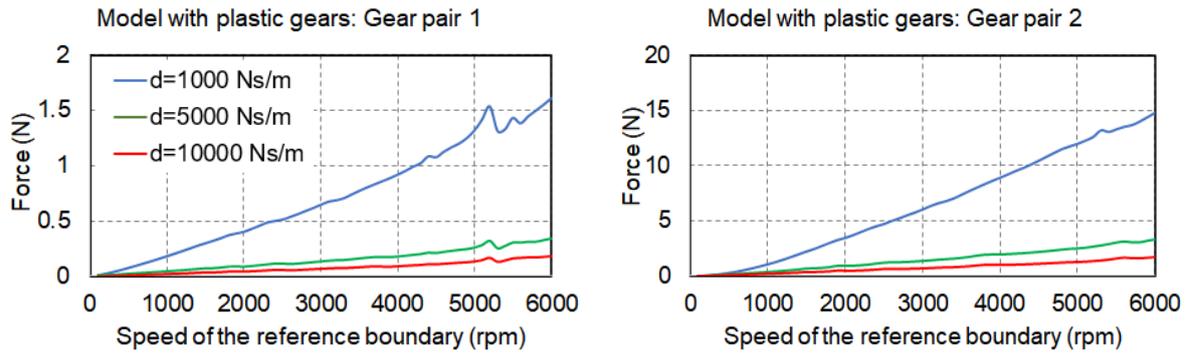


Figure 8: Double amplitudes of the dynamic meshing contact force of the plastic gears model with different gear meshing damping

To further investigate the effect of meshing damping, the double amplitudes of the dynamic reaction forces of bearings 5 and 6 (mounted on the output shaft) of the steel and plastic gears models with different gear meshing damping coefficients are plotted in Figs. 9 and 10, respectively.

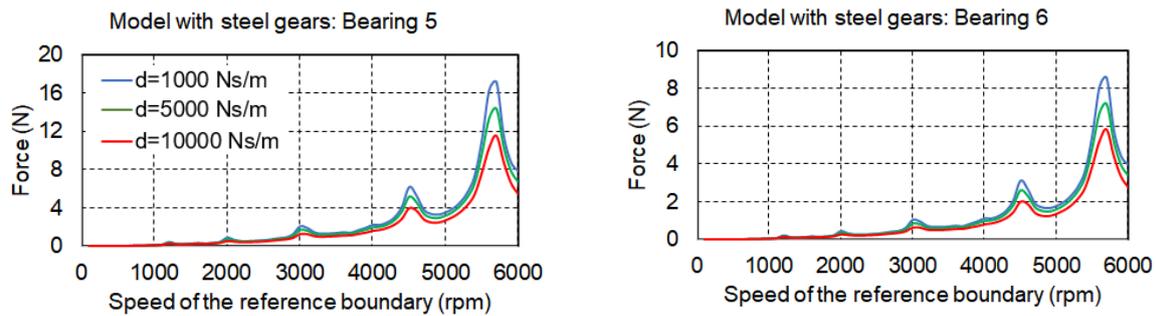


Figure 9: Double amplitudes of the dynamic bearing reaction forces of the steel gears model with different gear meshing damping

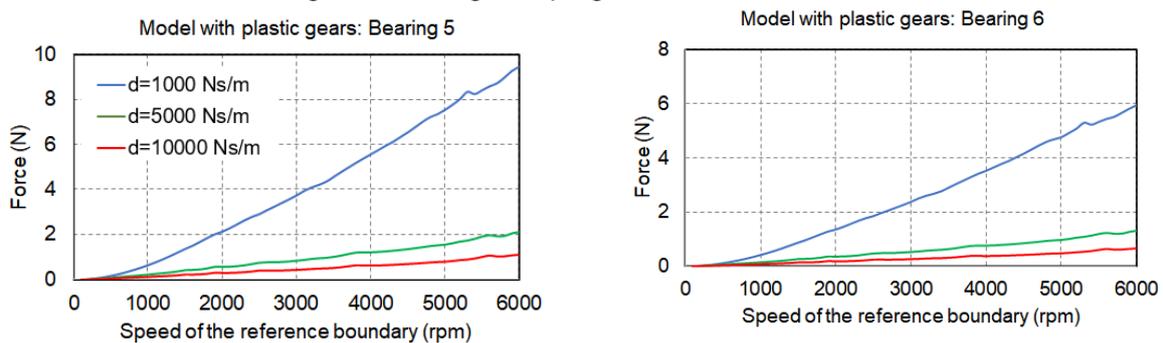


Figure 10: Double amplitudes of the dynamic bearing reaction forces of the plastic gears model with different gear meshing damping

By adding the meshing damping, a significant decrease in the model with plastic gears can be seen. These results clearly show that adding the gear meshing damping in the plastic gears model considerably reduces the fluctuations of the bearing forces which in turn can promise

improvement of the noise emission of the system. The main advantage of plotting the results versus the speed of the reference shaft is that it provides more insight into the response of the system in a wide range of input speeds. Accordingly, the speed of interest at which higher excitations can be perceived, is selected for further analysis. As it was already demonstrated in the results of the model with steel gears, the highest value of the double amplitudes of the meshing contact and bearing forces occur at a speed of 5600 rpm. It is well-known from the literature that plastic gears contribute more energy loss to the gear transmission system due to the higher values of the gear meshing damping as well as the structural damping. Consequently, the meshing damping of plastic gears is extremely higher than the steel gears. However, due to the complexity of the evaluation of the damping, no widely accepted model was presented [19]. Therefore, in the following steps, the meshing damping of the steel and plastic gears are, respectively, 1000 Ns/m and 5000 Ns/m to qualitatively compare the damping effect. For further analysis, the magnitude of the dynamic bearing reaction forces of bearings 5 and 6 are calculated and are shown in Fig. 11.

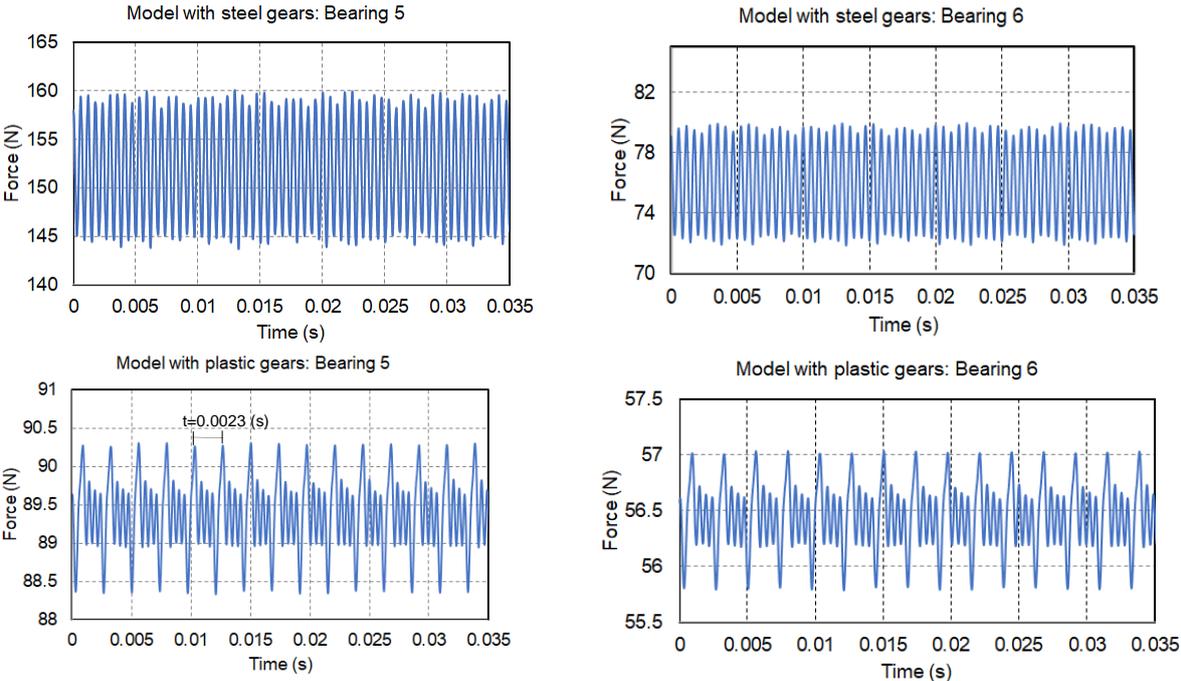


Figure 11: Magnitude of the dynamic bearing reaction forces for the speed of the reference boundary 5600 rpm, damping of the steel model  $d=1000$  Ns/m, damping of the plastic model  $d=5000$  Ns/m

The effect of damping on the bearing forces of the plastic gears model can be observed. The time interval between each two successive peaks of the bearing forces of the model with plastic gears is equal to the contact duration of a mating teeth of the second gear pair. This fact can

easily be verified by the gear mesh excitation orders reported in Table 2 for the second gear

$$\text{pair: } t = \frac{1}{5600/60 \cdot 4.55} = 0.0023 \text{ Seconds .}$$

#### 4. NVH analysis in RecurDyn based on the forced response analysis in KISSsoft

In previous sections, the forced response analysis of both gear transmission models with steel and plastic gears in KISSsoft was presented. As the result of the analysis, the bearing forces using different gear meshing damping factors were calculated. In the next step, the bearing forces are imported to RecurDyn [20] and applied directly to the housing of the powertrain model at bearing positions. The bearings, shafts and gears are not included in the model. The analysis starts with setting the parameters of the housing and extracting a reduced order flexible model in a modal analysis approach by selecting the first 50 eigenfrequencies. Then, the bearing forces are applied as time-dependent spline functions at the bearing position of the housing. Before conducting any NVH analysis, the housing responses are calculated in a dynamic analysis. As a major kinematic parameter required for the NVH analysis, the surface velocities at nodes of the meshed geometry are calculated. Finally, the equivalent radiated power  $ERP$ , as the main factor for measuring the emitted noise level from the housing's surface to the environment is calculated as follows:

$$ERP = f_{RLF} \cdot \frac{1}{2} \cdot C \cdot \rho \cdot \sum (A_i \cdot v_i^2)$$

where  $f_{RLF}$  is the radiation loss factor,  $C$  is sound velocity,  $\rho$  is the density of a target material which transfers the noise, e.g., air,  $A_i$  is the area on the  $i^{th}$  flexible panel of the meshed surface, and  $v_i$  is its face normal velocity. Further details can be found in [20]. To clearly demonstrate which parts of the housing's surface emit higher noise, the contour plot of the  $ERP$  is very helpful. It can subsequently be used to address demanded design modifications, such as local stiffening of the housing by means of the ribs, to reduce the noise. Figure 12 shows  $ERP$  contours of both models using the bearing reaction forces calculated in the forced response at a speed of 5600 rpm.  $T_{max}$  denotes the time at which the  $ERP$  reaches its maximum value. It is clearly seen that in both models, the region of the housing close to the output shaft has higher  $ERP$  values, and consequently, emits noise to the environment. In addition, the plastic gears model has a superior performance with respect to the noise emission. The difference between the scales of the  $ERP$  of both models reveals this superiority. The maximum values of the color legends are adjusted to clearly show the noise distribution through the housing's surface. It is noticeable to mention that the variation of the  $ERP$  depends on the simulation time and on the locally evaluated position on the housing.

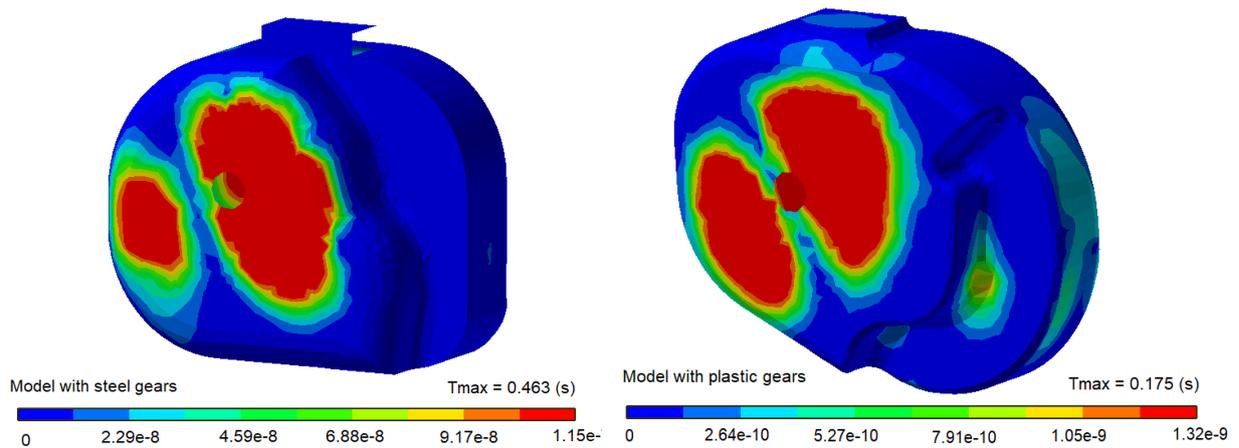


Figure 12: ERP contours at the time with maximum values at speed of 5600 rpm

To further clarify the performance of both models, the *ERP* in time and frequency domains are calculated. The results are plotted in Figs. 13 and 14 for the nominal speed of 2742 rpm and in Figs. 15 and 16 for the speed of 5600 rpm. According to the results, the higher contribution of the steel gears model compared to the plastic gears model in the noise generation can be observed at speed 5600 rpm. However, the difference of the *ERP* between both models at speed 2742 rpm is not significant.

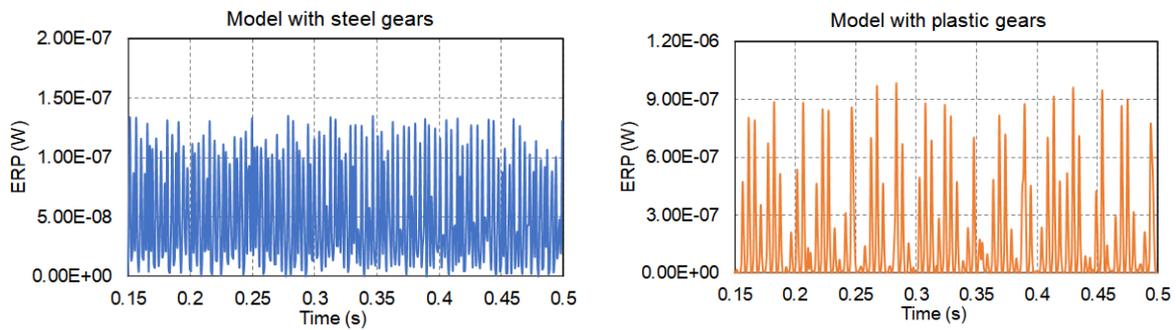


Figure 13: Magnitude of the ERP versus time for the speed of 2742 rpm, damping of the steel model  $d=1000$  Ns/m, damping of the plastic model  $d=5000$  Ns/m

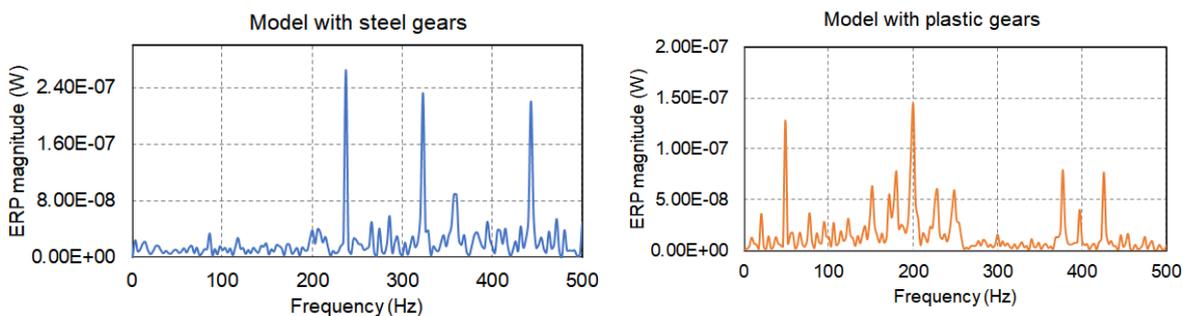


Figure 14: Magnitude of the ERP versus frequency for the speed of 2742 rpm, damping of the steel model  $d=1000$  Ns/m, damping of the plastic model  $d=5000$  Ns/m

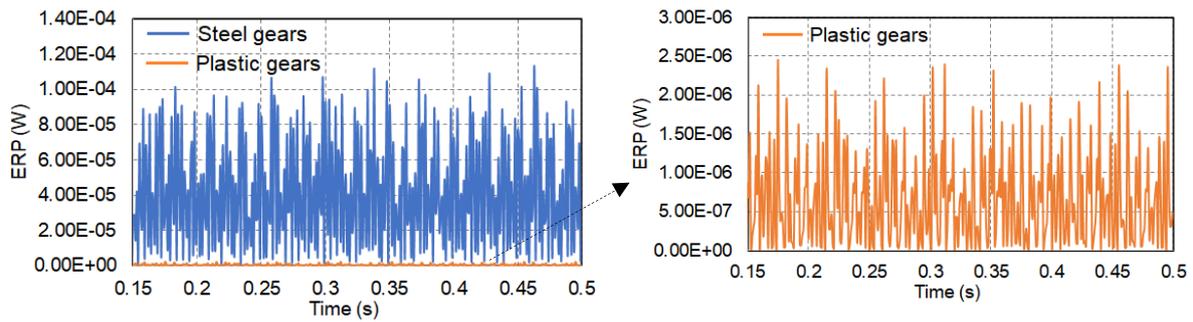


Figure 15: Magnitude of the ERP versus time for the speed of 5600 rpm, damping of the steel model  $d=1000$  Ns/m, damping of the plastic model  $d=5000$  Ns/m

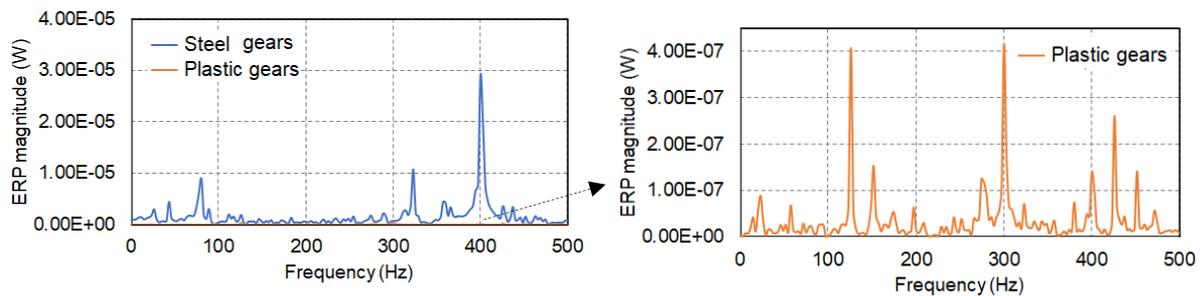


Figure 16: Magnitude of the ERP versus frequency for the speed of 5600 rpm, damping of the steel model  $d=1000$  Ns/m, damping of the plastic model  $d=5000$  Ns/m

These results are in agreement with the double amplitudes of the dynamic bearing reaction forces in Figs. 9 and 10. These results confirm that proper selection of the gears with desirable damping properties lead to a lower level of noise emission by reducing the double amplitudes of the excitation forces and subsequently lower excitations to the powertrain system at critical speeds.

## 5. Conclusion:

This paper presented some steps to get more insight into the effect of meshing damping on the dynamic response and noise emission of a two-stage gearbox transmission system. For this purpose, two variants of the model with different gear materials, one with plastic gears and one with steel gears, were considered. Both variants were designed for the same number of cycles of operation with similar root and flank safeties. The forced response analysis of the models was carried out and the exciting reaction bearing forces were calculated to evaluate which model can achieve better NVH characteristics results with lower noise emission from the housing. The results showed that the damping considerably decreases the dynamic factors and the variation of the bearing reaction forces of the gearbox model with plastic gears. It was observed that the effect of damping at lower values for plastic gears is dominant, which means that the reduction of the dynamic factor from  $d=0$  Ns/m to 1000 Ns/m is higher than its reduction

from 1000 Ns/m to 5000 Ns/m. The same conclusion can be drawn by comparing the dynamic factor reduction from successive damping increase. It was observed that for plastic gears, the damping should not be neglected even when using an approximate damping is possible, otherwise it would lead to the unrealistic response of the system. Therefore, for plastic gears, consideration of meshing damping by selecting suitable gear material can be an efficient approach for damping undesirable induced vibrations. On the other hand, for steel gears, it is not crucial as plastic gears to use damping, since the results are not much affected when compared to the plastic gears. This process can result in lower noise emission, decrease dynamic loads and potential damages of the gear teeth and bearings. Further investigation of the models in RecrDyn based on the forced response analysis results confirmed that the plastic gears model has a superior performance with respect to the noise emission from the housing's surface.

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