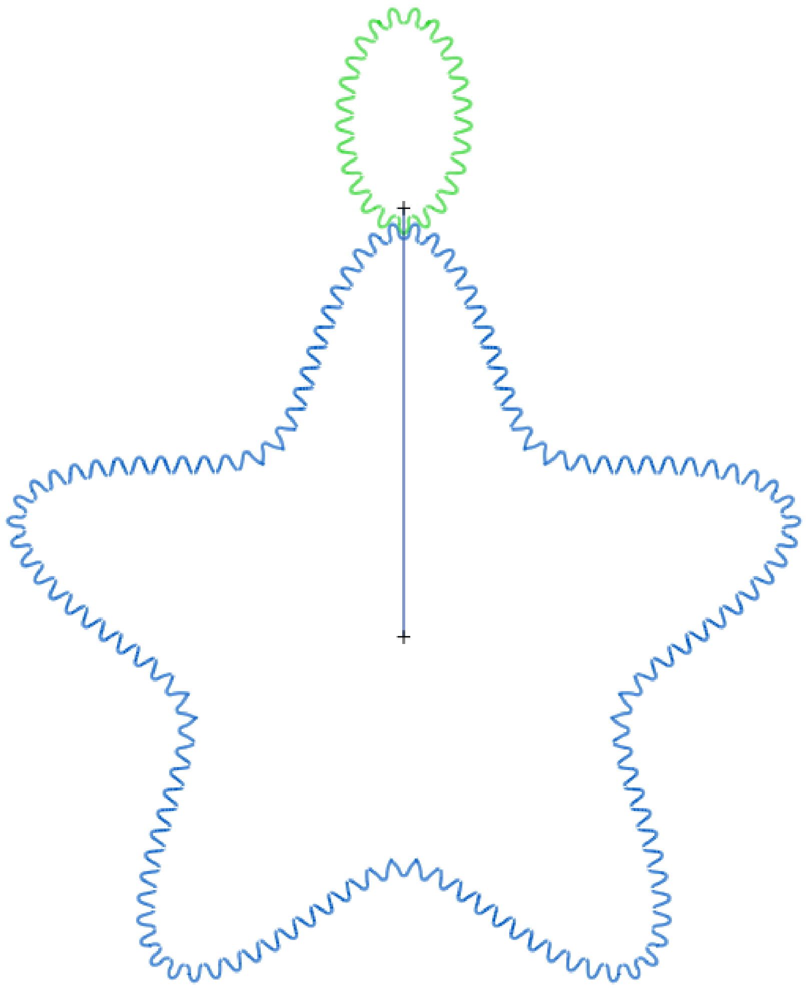
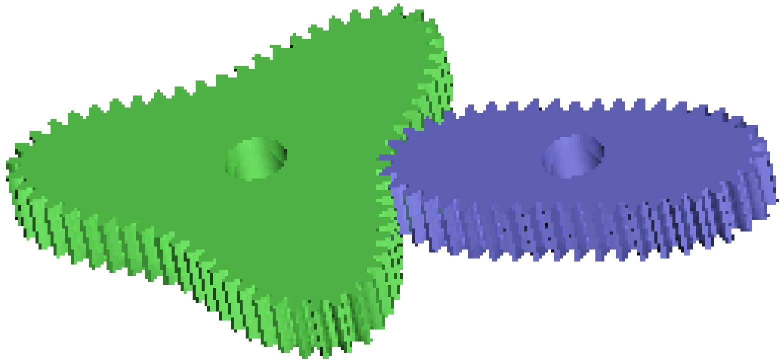
Noncircular gears
Theory and applications



Dr. Aljaz Pogacnik

Content

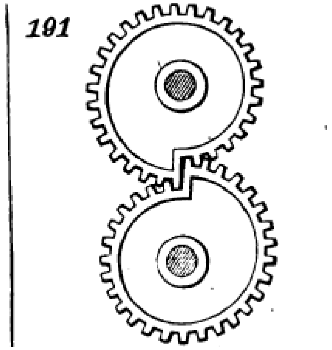
- Introduction
- Design principles
- Design possibilities
- Generation of tooth form
- Strength calculation
- Noncircular gears in KISSsoft
- Case studies

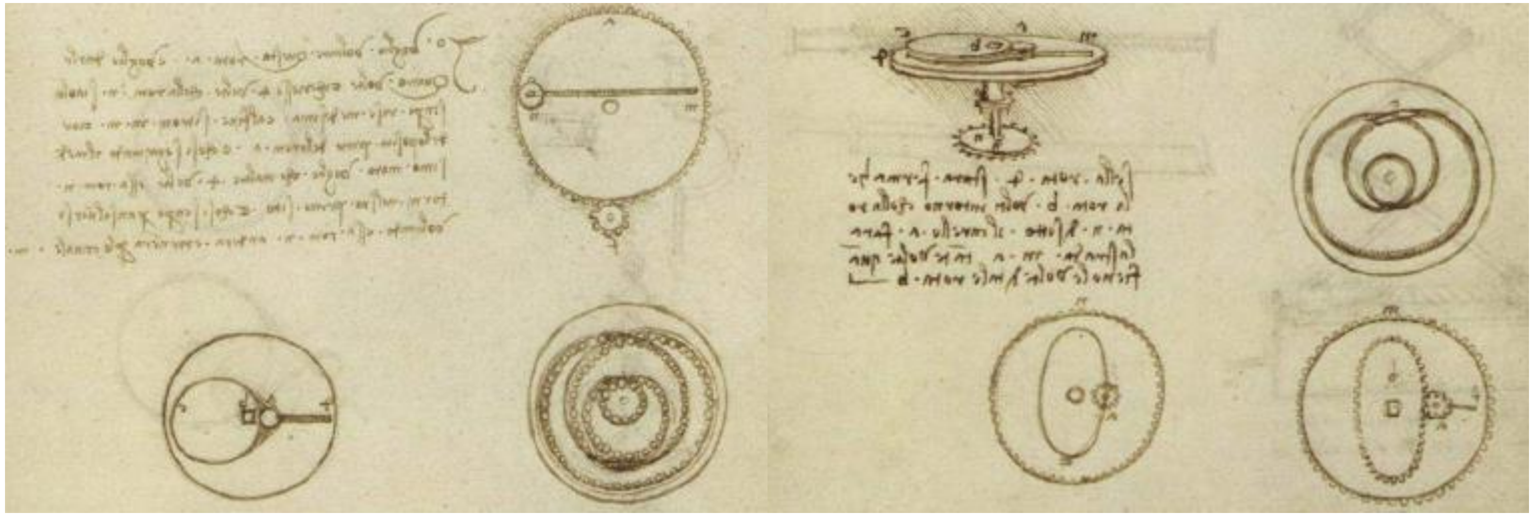


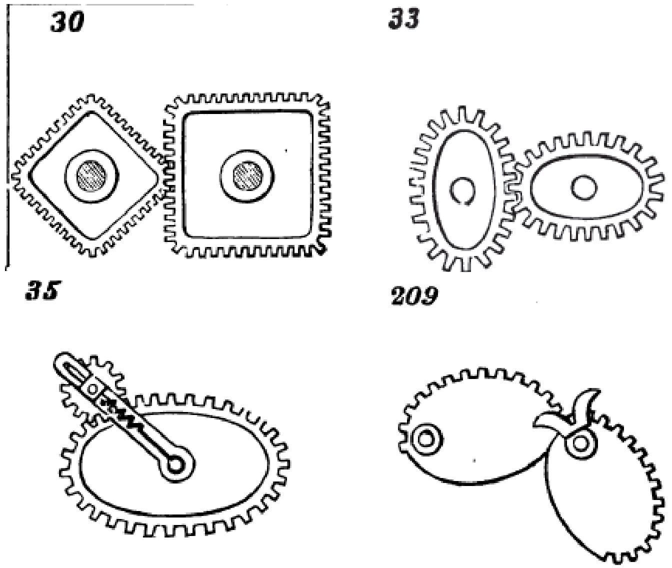
GIF source: http://www.gearandrack.com/non_circular_gears/simulation_test_for_non_circular_gears.html

Introduction

- Noncircular gears (NCGs) first sketched by Leonardo da Vinci (1500)
- First known publications came in the 19th century
- A comprehensive overview of the non-circular gears was done by Litvin
- Limited data available especially compared to cylindrical gears







Picture source: H.T. Brown, Five Hundred and Seven Mechanical Movements

Picture source: B. Laczik, Design and Manufacturing of Non-Circular Gears by Given Transfer Function

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Introduction

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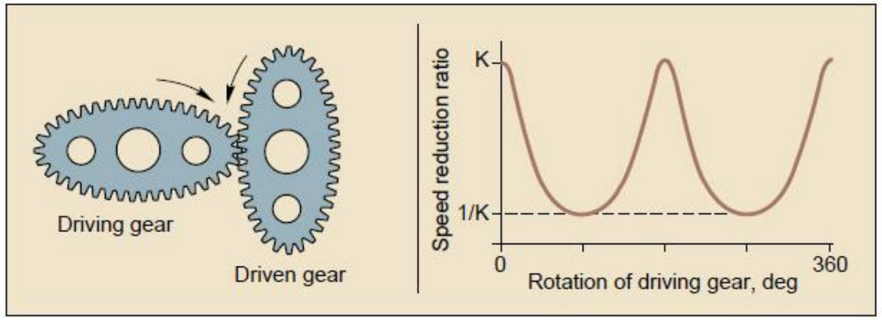
Advantages

- Variable transmission ratio
- Variable speed and torque
- Variable center distance
- Manufacturing process (if plastic or sinter)

Limitations

- Vibrations if not rotating in mass centre
- Strength calculation
- Manufacturing process (if steel)



Picture source: www.machinedesign.com/motorsdrives/noncircular-gears-make-unconventional-moves

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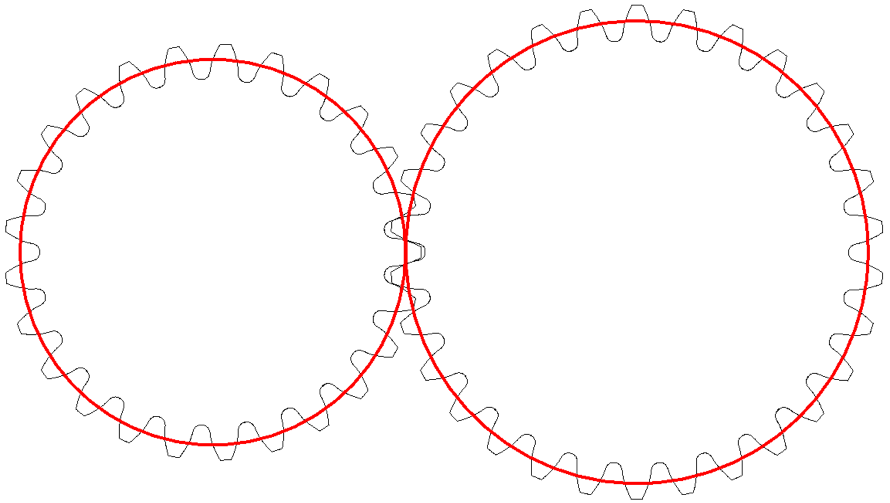
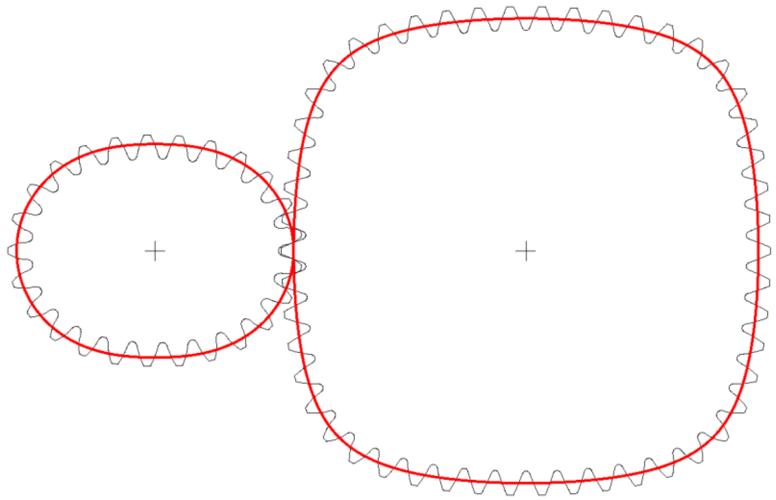
Design principles

- **Cylindrical gears:** operating pitch lines are circles



- **Non-circular gears:** operating pitch lines are NOT circles



According to the law of gear meshing, the operating pitch lines must roll together without slip!

Picture source: www.machinedesign.com/motorsdrives/noncircular-gears-make-unconventional-moves

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Design principles

- The **operating pitch lines** can be described by $r_1(\varphi_1)$ and $r_2(\varphi_2)$.

- The **transmission ratio** function can be written as

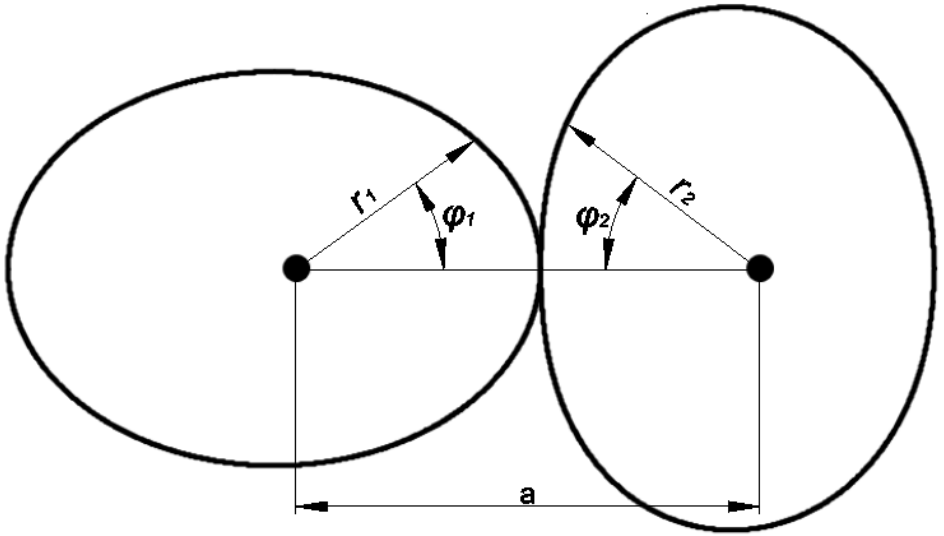
$$i(\varphi_1) = \frac{\omega_1(\varphi_1)}{\omega_2(\varphi_2)} = \frac{r_2(\varphi_2)}{r_1(\varphi_1)} \quad \textit{(eq. 1)}$$

- The **center distance** is defined as

$$r_1(\varphi_1) + r_2(\varphi_2) = a = const \qquad \textit{(eq. 2)}$$

- Eq. 1 can be rewritten to

$$i(\varphi_1) = \frac{a - r_1(\varphi_1)}{r_1(\varphi_1)} \qquad \textit{(eq. 3)}$$



Design principles

- **Law of gearing:** operating pitch lines must roll without slip

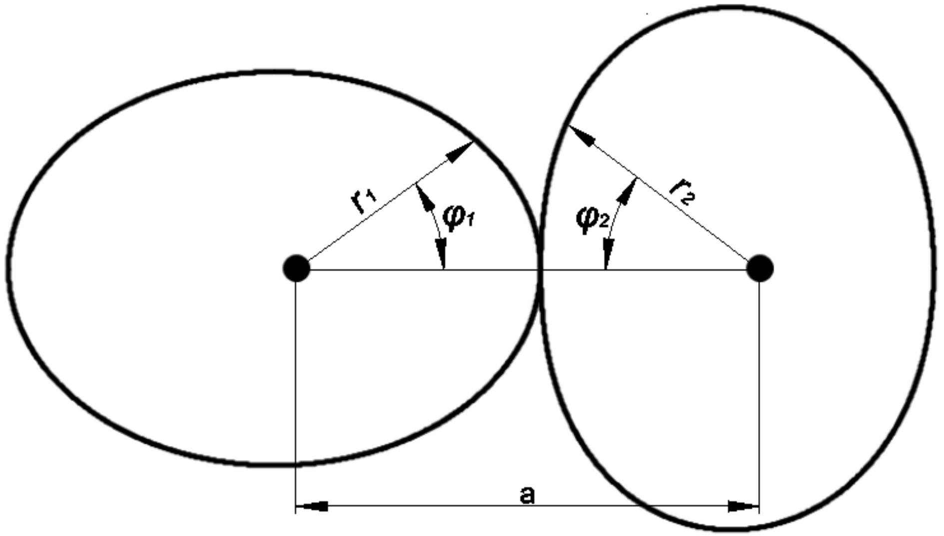
$$r_1(\varphi_1)d\varphi_1 = r_2(\varphi_2)d\varphi_2$$

$$d\varphi_2 = \frac{r_1(\varphi_1)}{r_2(\varphi_2)}d\varphi_1 \qquad \textit{(eq. 4)}$$

- **Calculation case 1**: $r_1(\varphi_1)$ is known

$$d\varphi_2 = \frac{r_1(\varphi_1)}{a - r_1(\varphi_1)}\, d\varphi_1$$

$$\varphi_2 = \int_0^{\varphi_1} \frac{r_1(\varphi_1)}{a - r_1(\varphi_1)}\, d\varphi_1$$



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Design principles

- **Law of gearing:** operating pitch lines must roll without slip

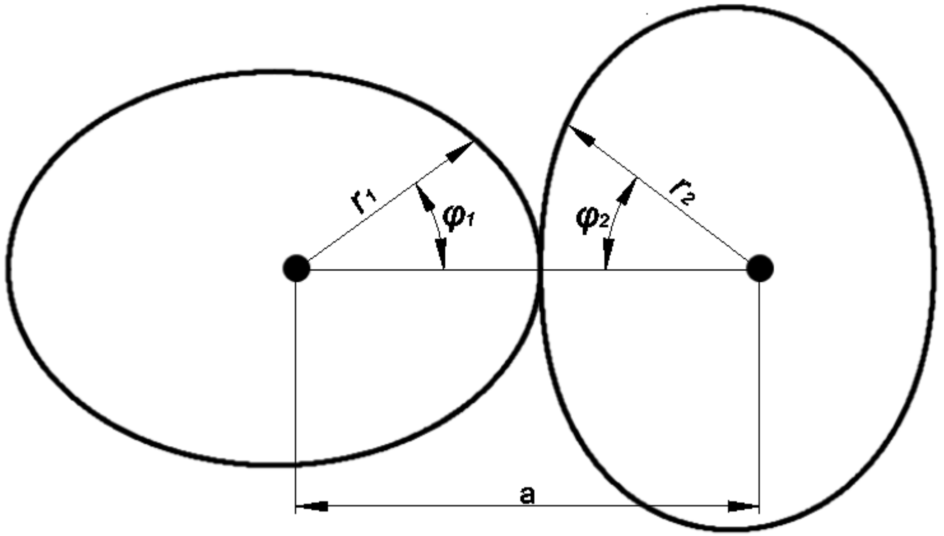
$$r_1(\varphi_1)d\varphi_1 = r_2(\varphi_2)d\varphi_2$$

$$d\varphi_2 = \frac{r_1(\varphi_1)}{r_2(\varphi_2)}d\varphi_1 = \frac{1}{i(\varphi_1)}d\varphi_1$$

- **Calculation case 2**: $i(\varphi_1)$ is known

$$d\varphi_2 = \frac{1}{i(\varphi_1)}\,d\varphi_1$$

$$\varphi_2 = \int_0^{\varphi_1} \frac{1}{i(\varphi_1)}\,d\varphi_1$$



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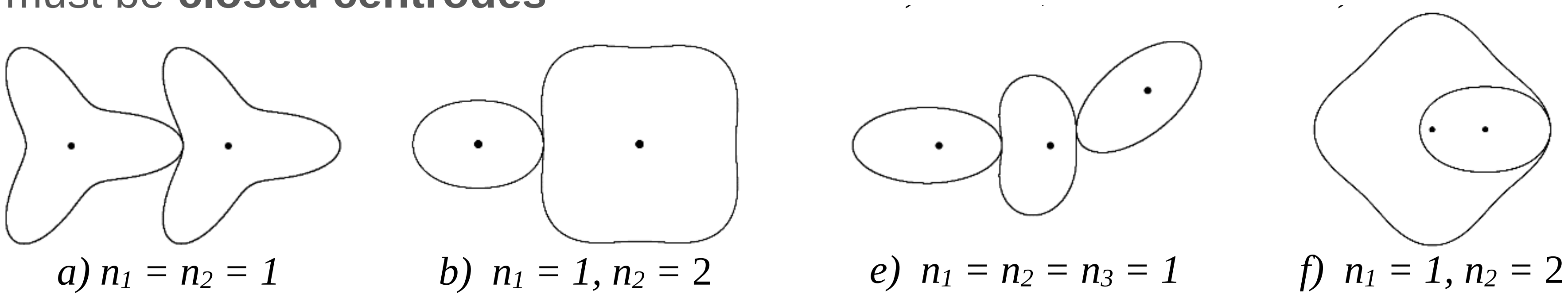
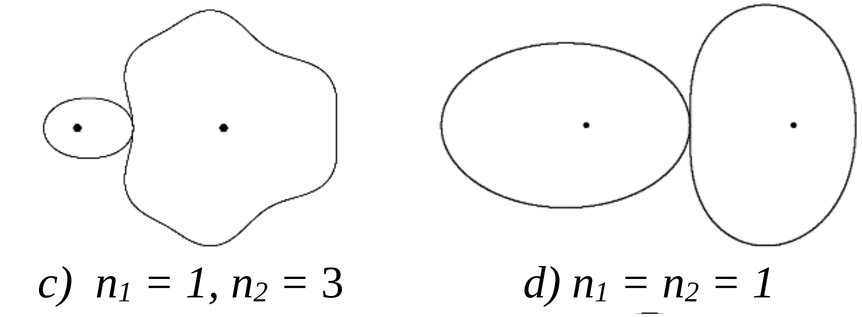
Design principles

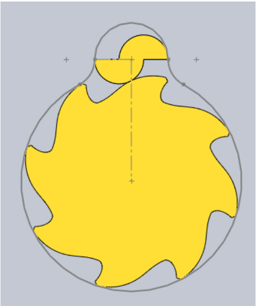
- To have **continuous rotation** of the non-circular gears, the operating pitch lines must be **closed centrodes**

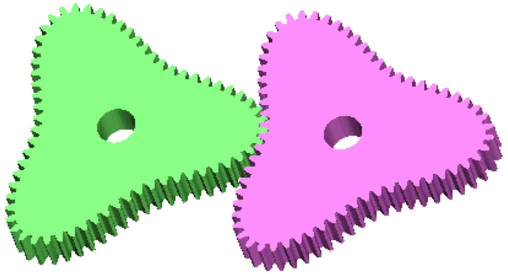


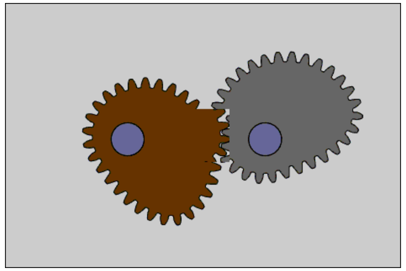


$$\frac{T_1}{n_2} = \frac{T_2(period)}{n_1(revolution)}$$

Where n_1 and n_2 are Z^+.

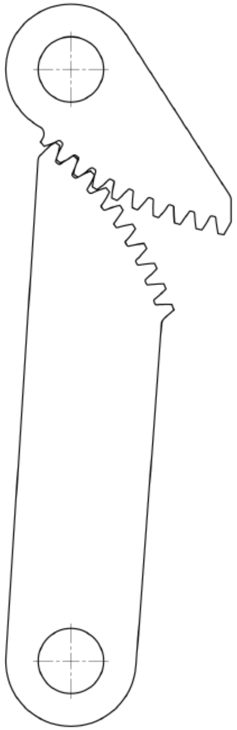




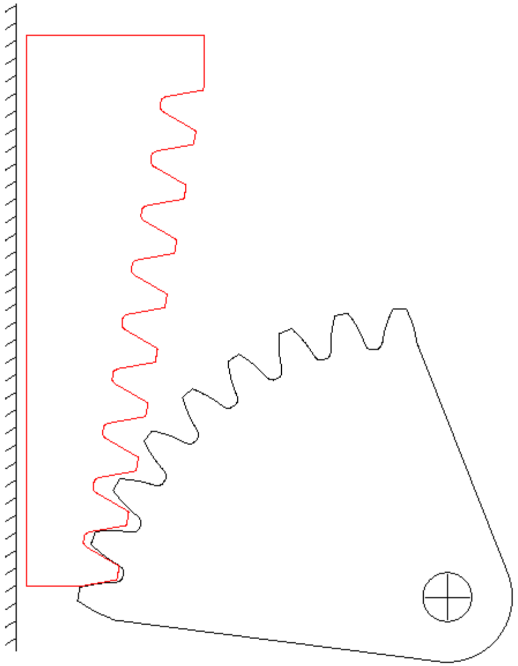


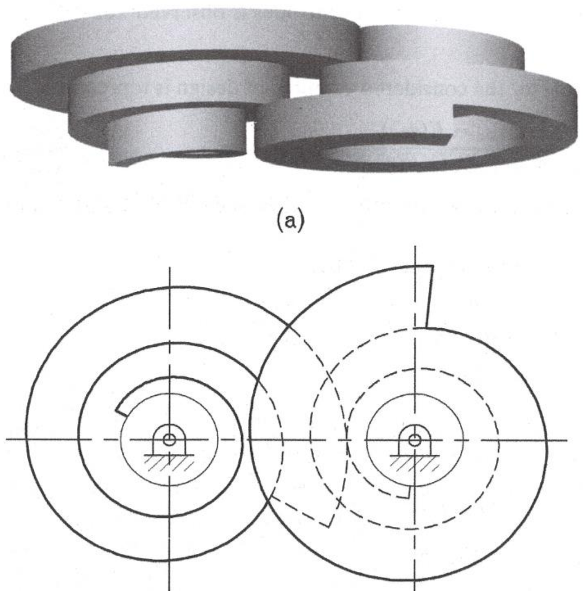
Design principles

- In case of **unclosed centrodes**, the angular rotation (or lateral movement) of the engaging gears is limited



[1]





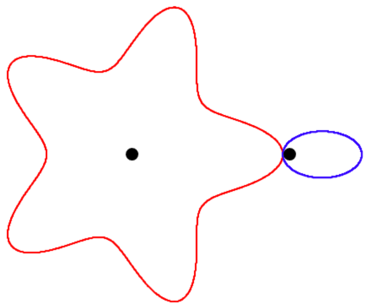
Twisted non circular gears

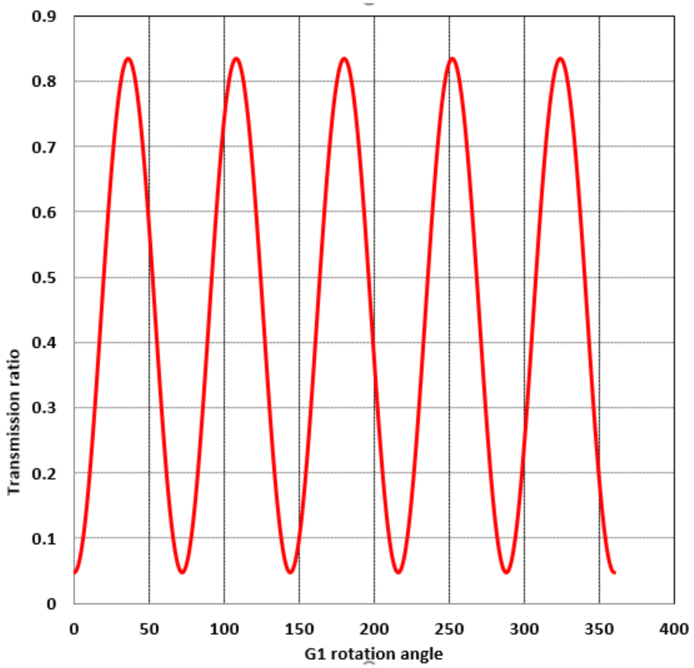
[2]

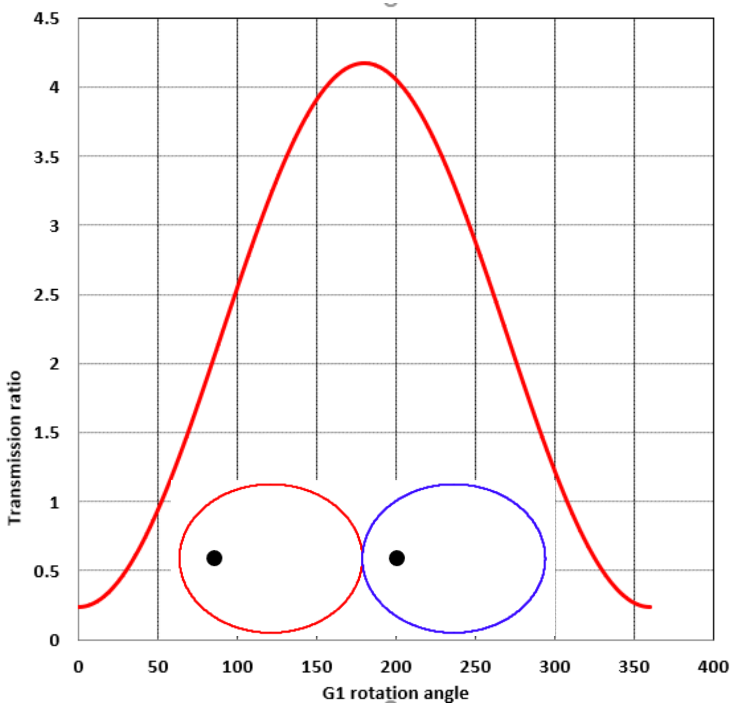
Picture source: [1] Zarebski I, Salacinski T., Designing of Non-Circular Gears
[2] Litvin F., Non Circular gears (Design and generation)

Design principles

- **Curves used for operating pitch lines**
  - Ellipses
  - N-lobbed ellipses
  - Circles with eccentricity
  - Arbitrary curves
- Additional eccentricity
- Variable center distance



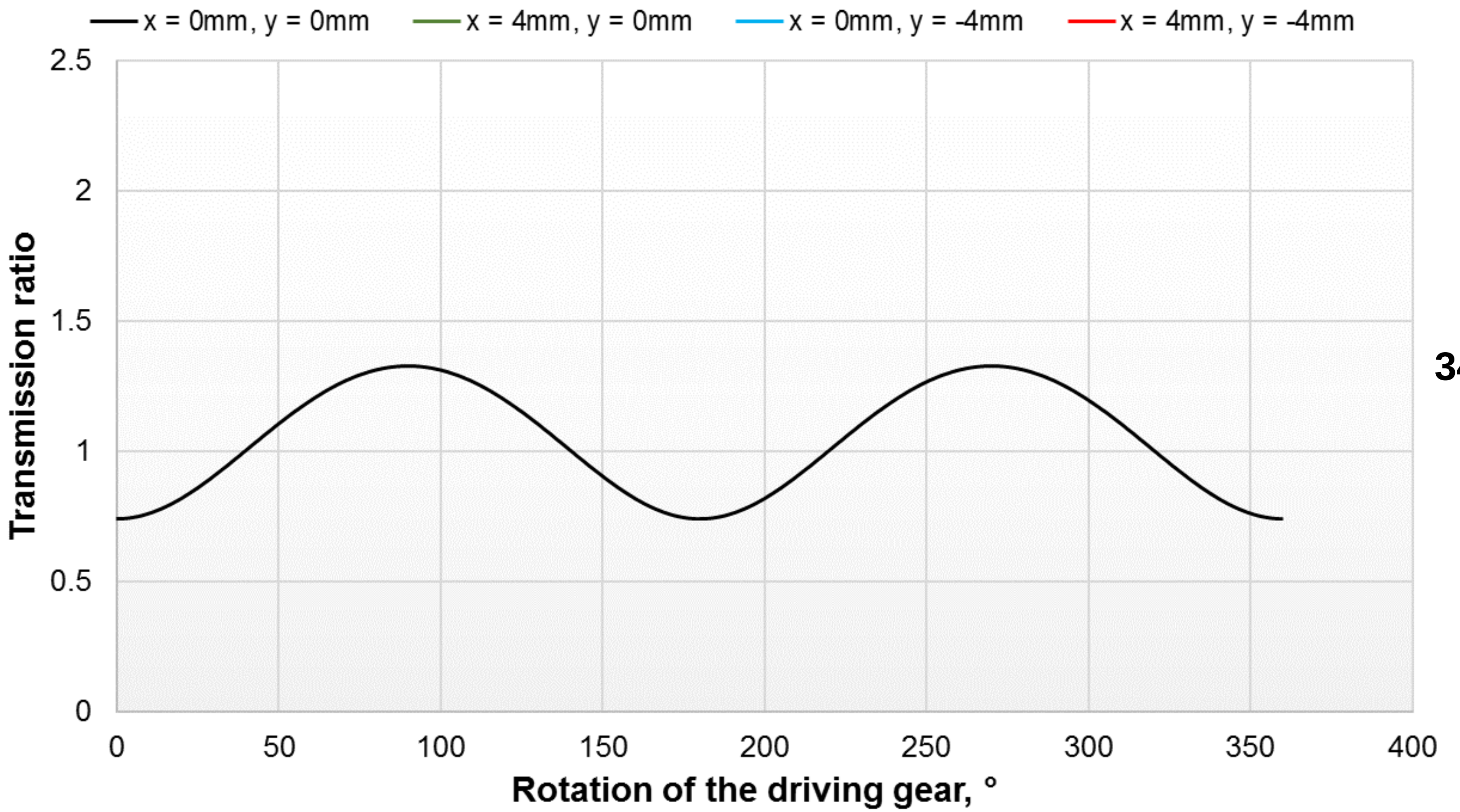


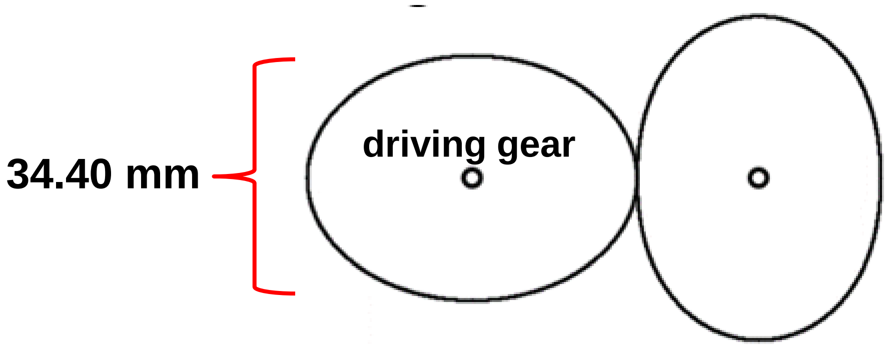


Vast number of different design possibilities!

Effect of eccentricity

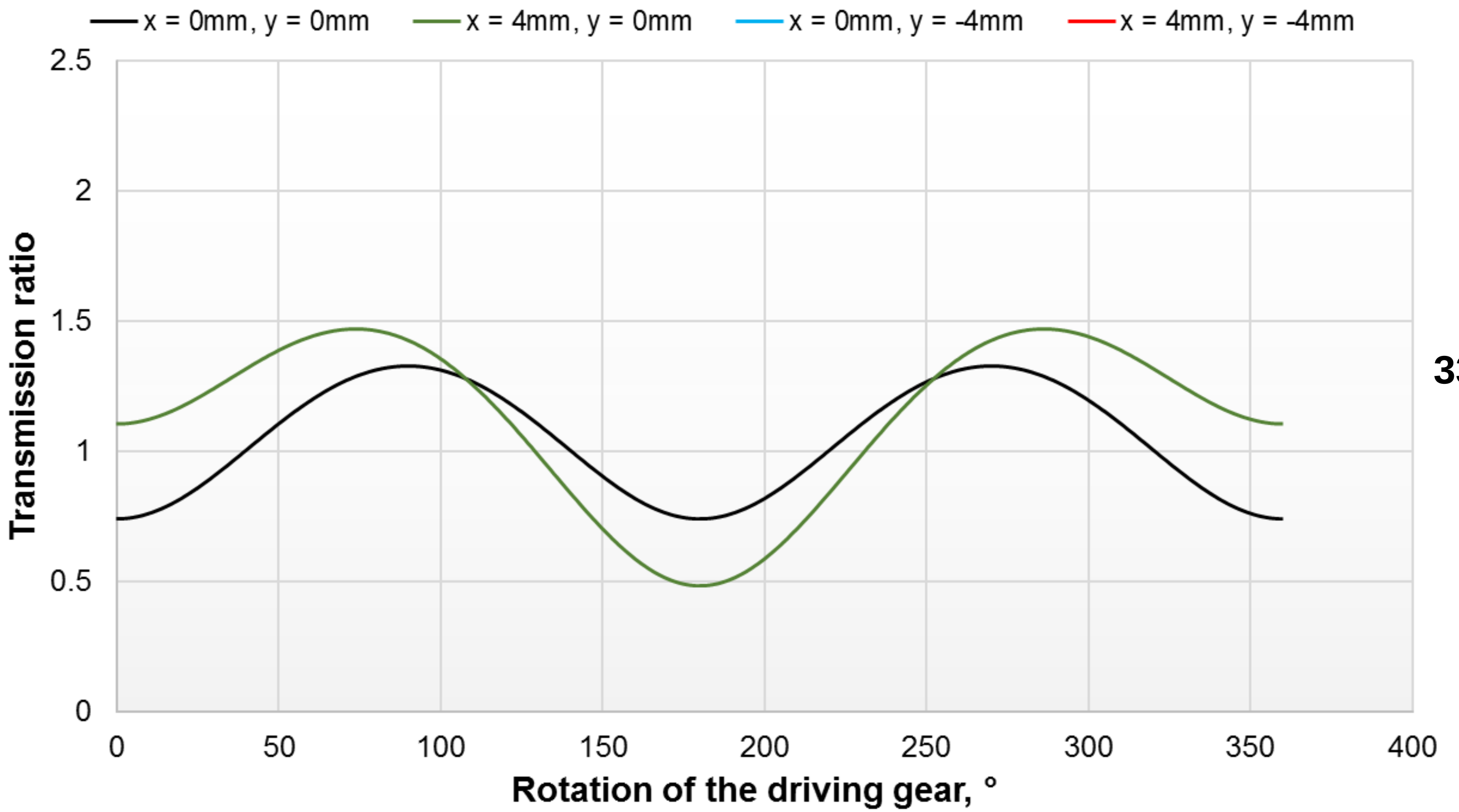
Fixed center distance: 40 mm

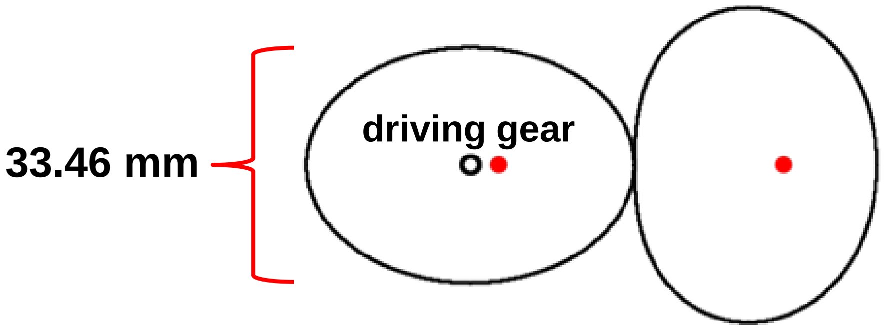




Effect of eccentricity

Fixed center distance: 40 mm





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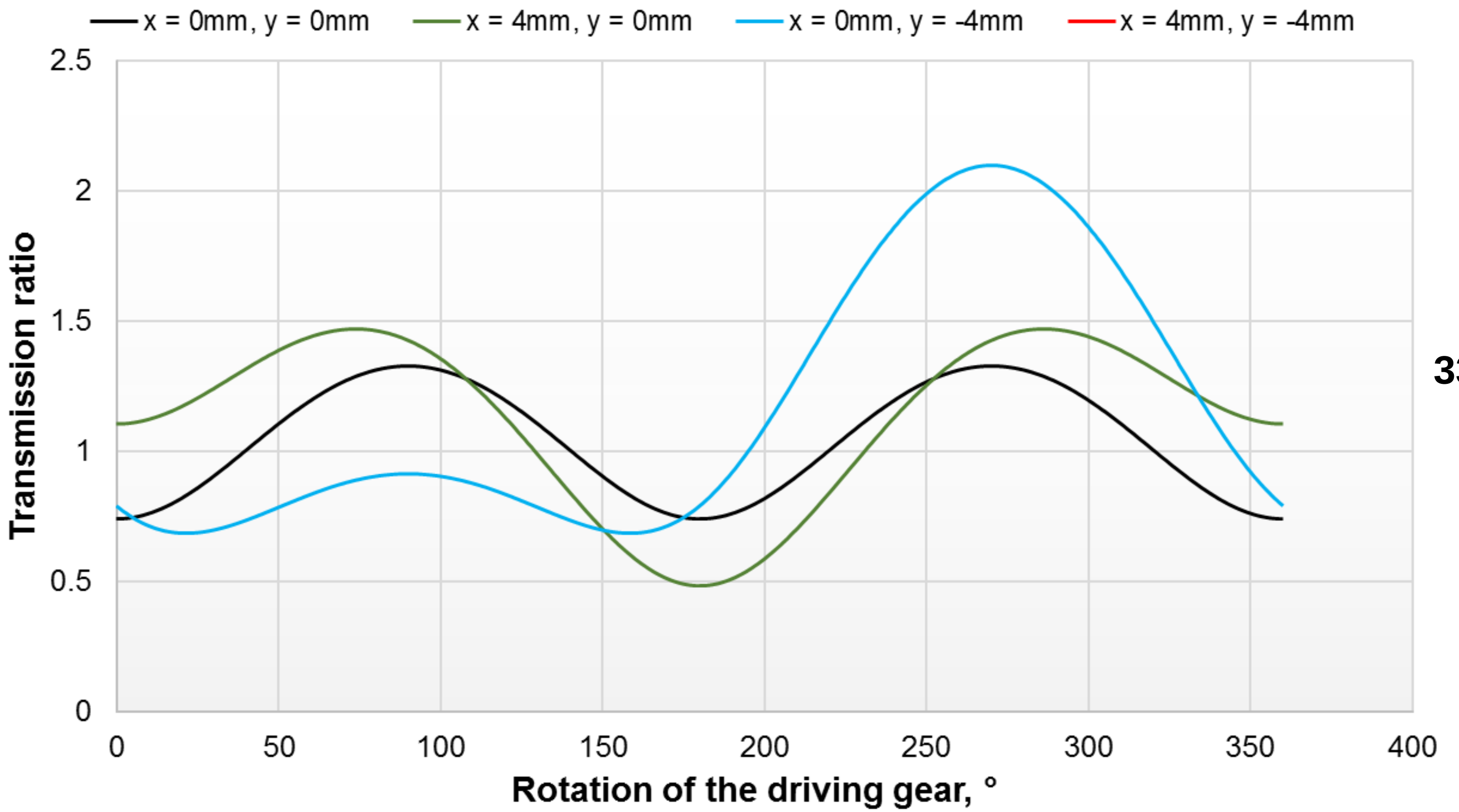
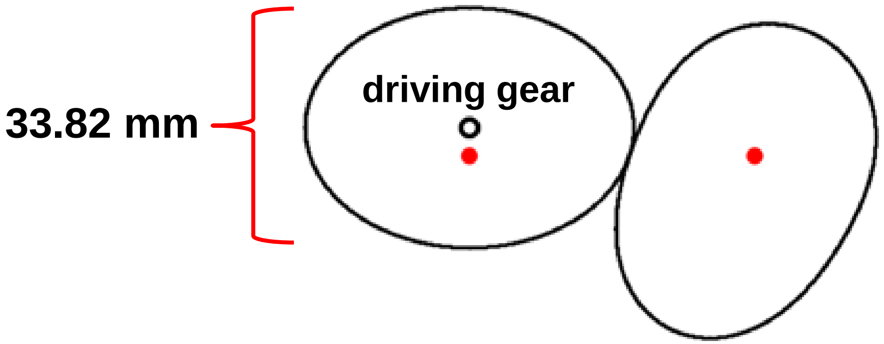
Effect of eccentricity

Fixed center distance: 40 mm





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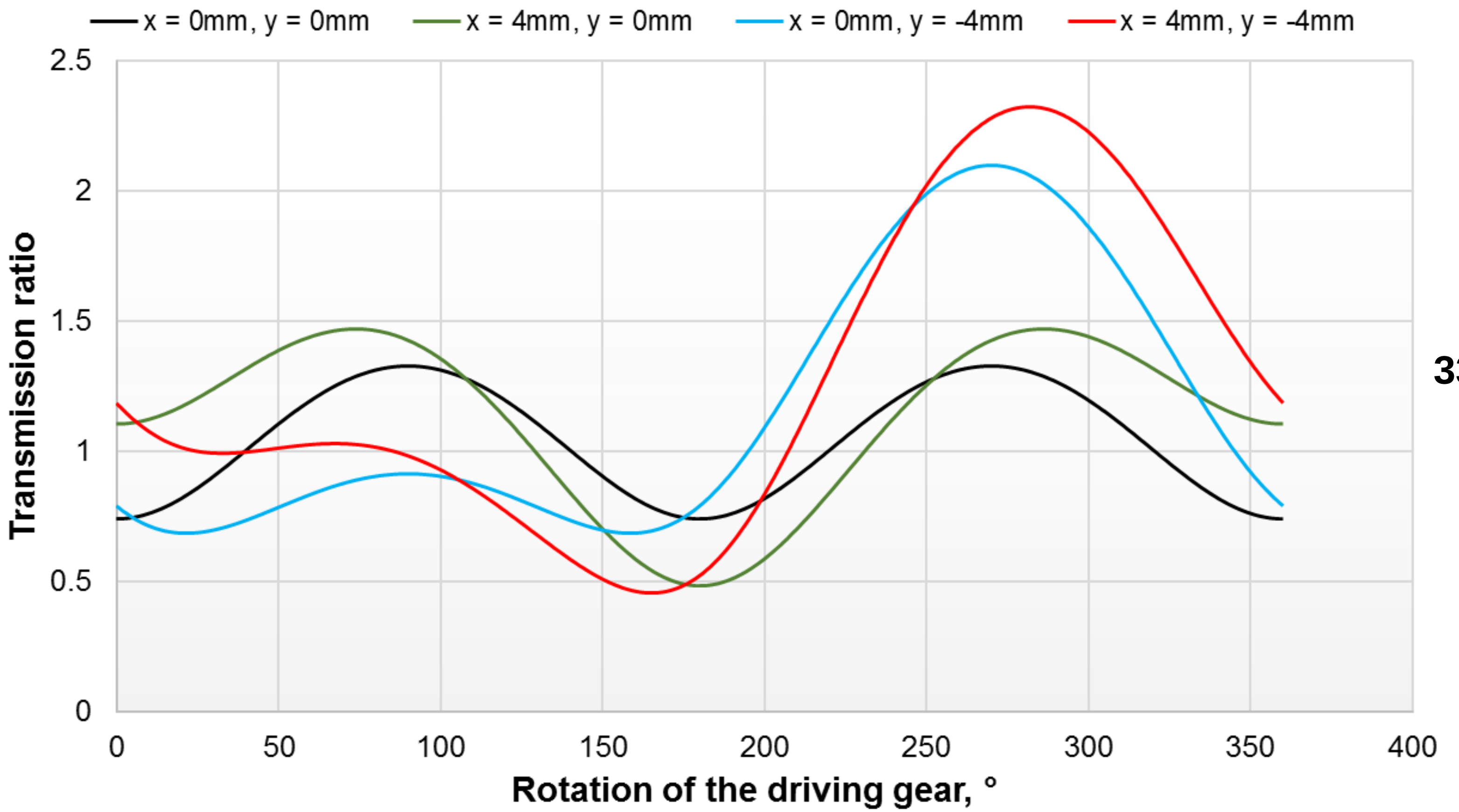
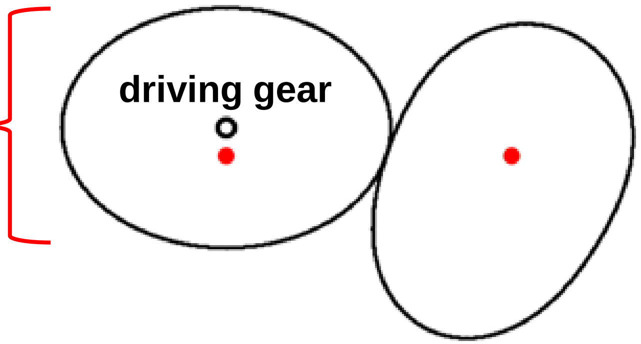
Effect of eccentricity

Fixed center distance: 40 mm



33.82 mm



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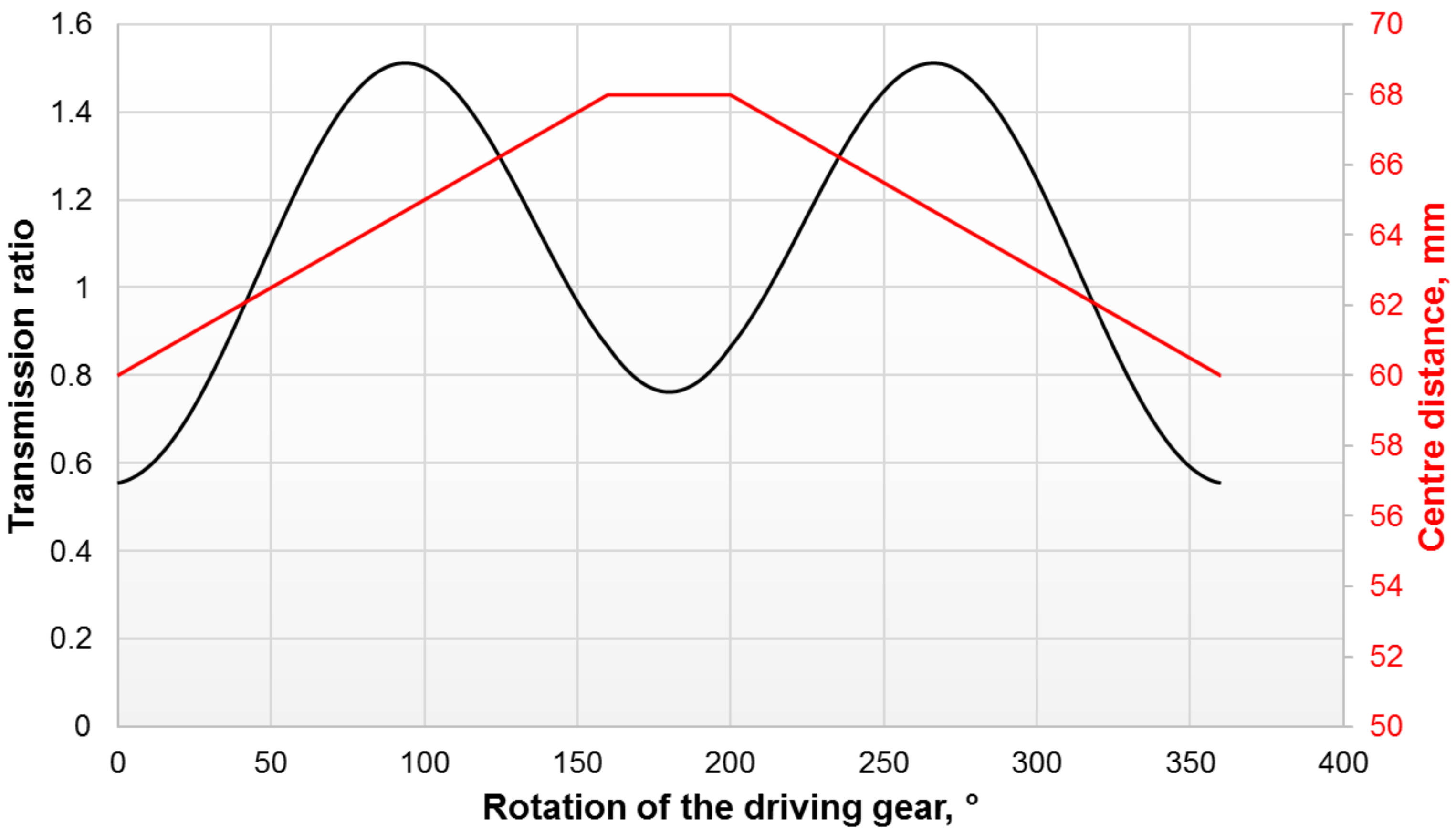
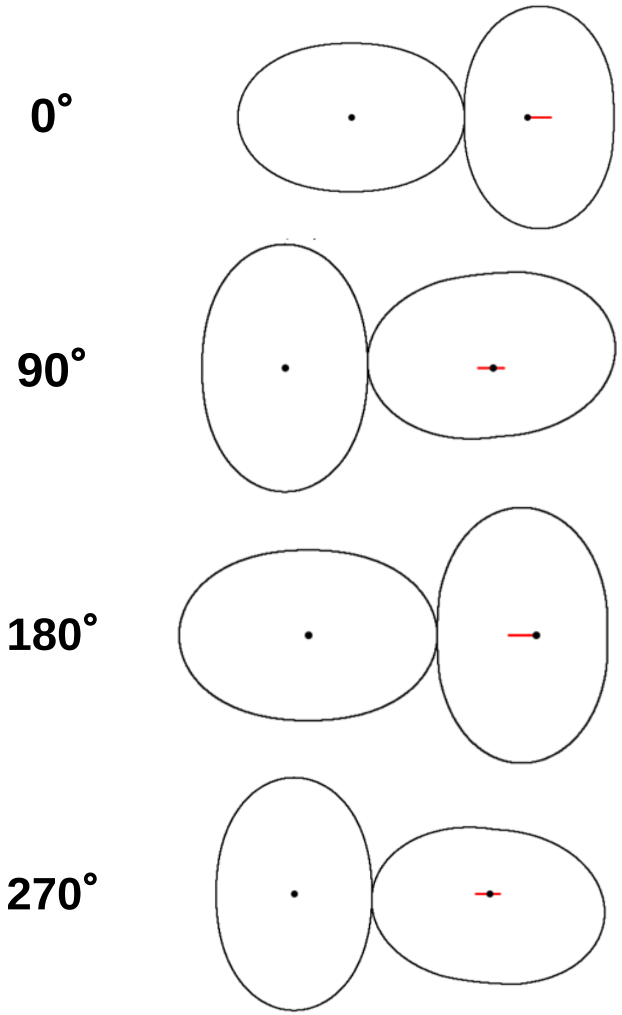
Variation of center distance





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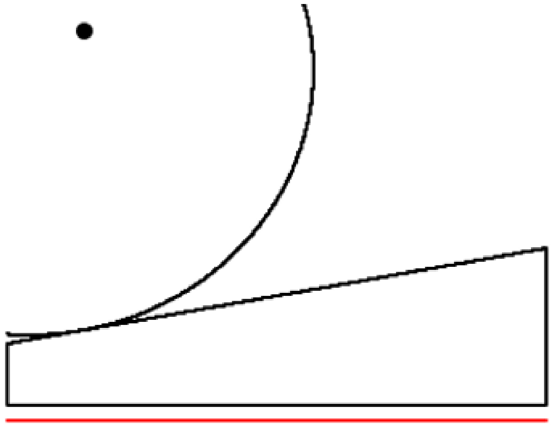
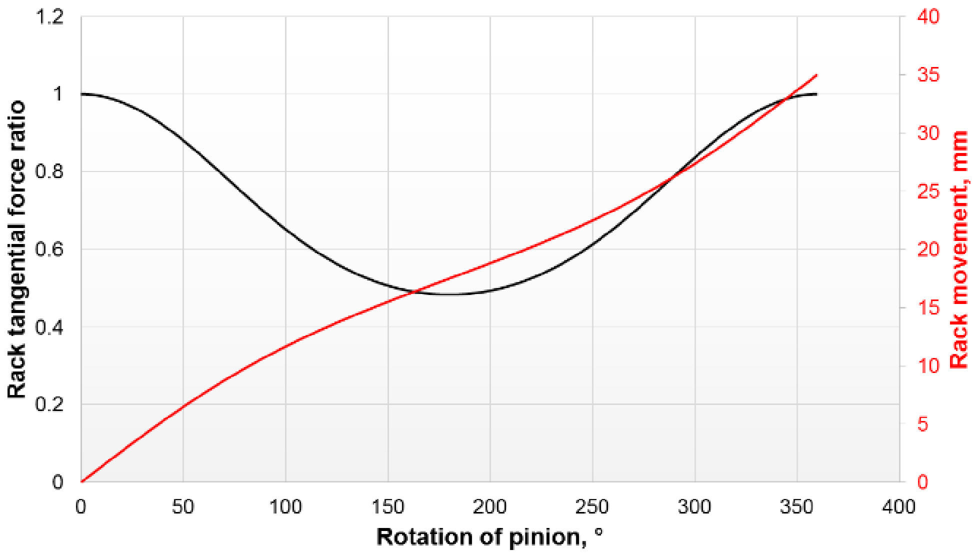
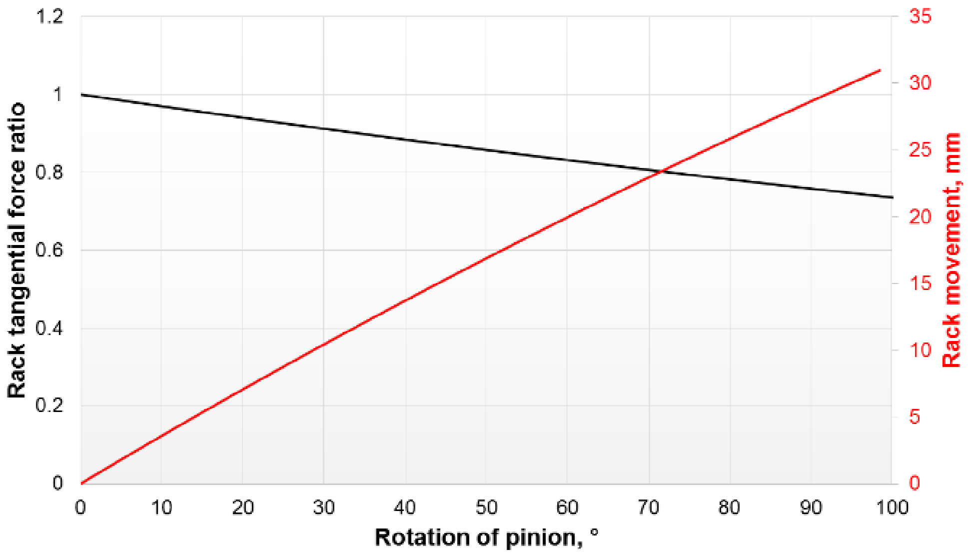
Rack and pinion design







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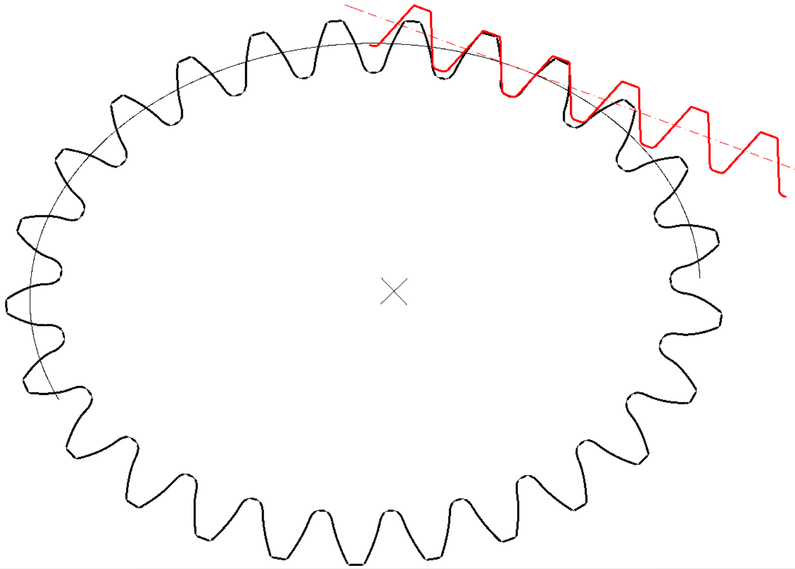
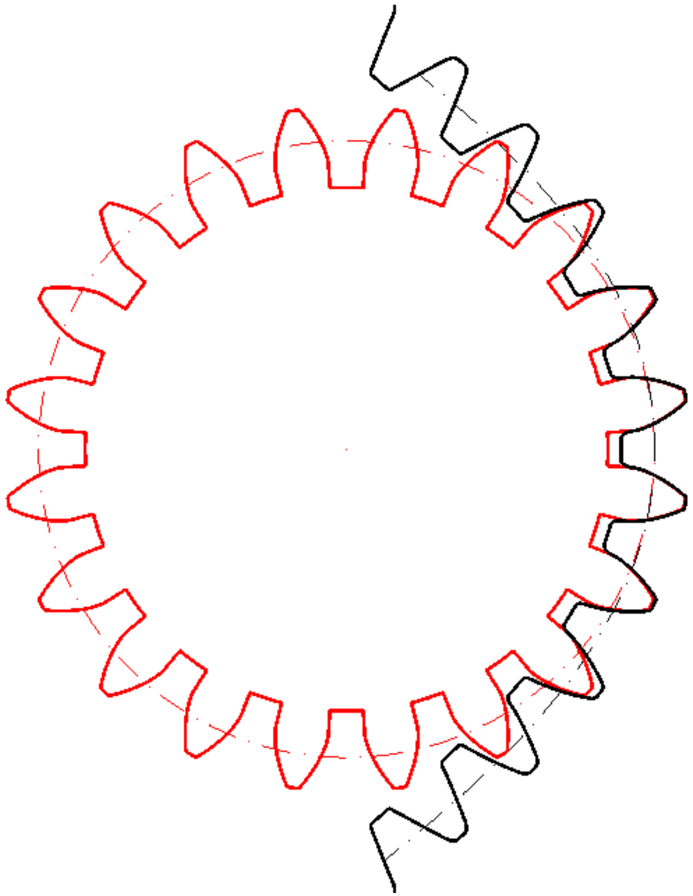
Generation of tooth form

The tooth form of the noncircular gears can be generated by a reference rack. In order to generate the tooth form, the reference lines must roll without slip.

To produce the inner noncircular gears, a pinion type cutter must be used. A pinion type cutter can also be used to produce outer gears.

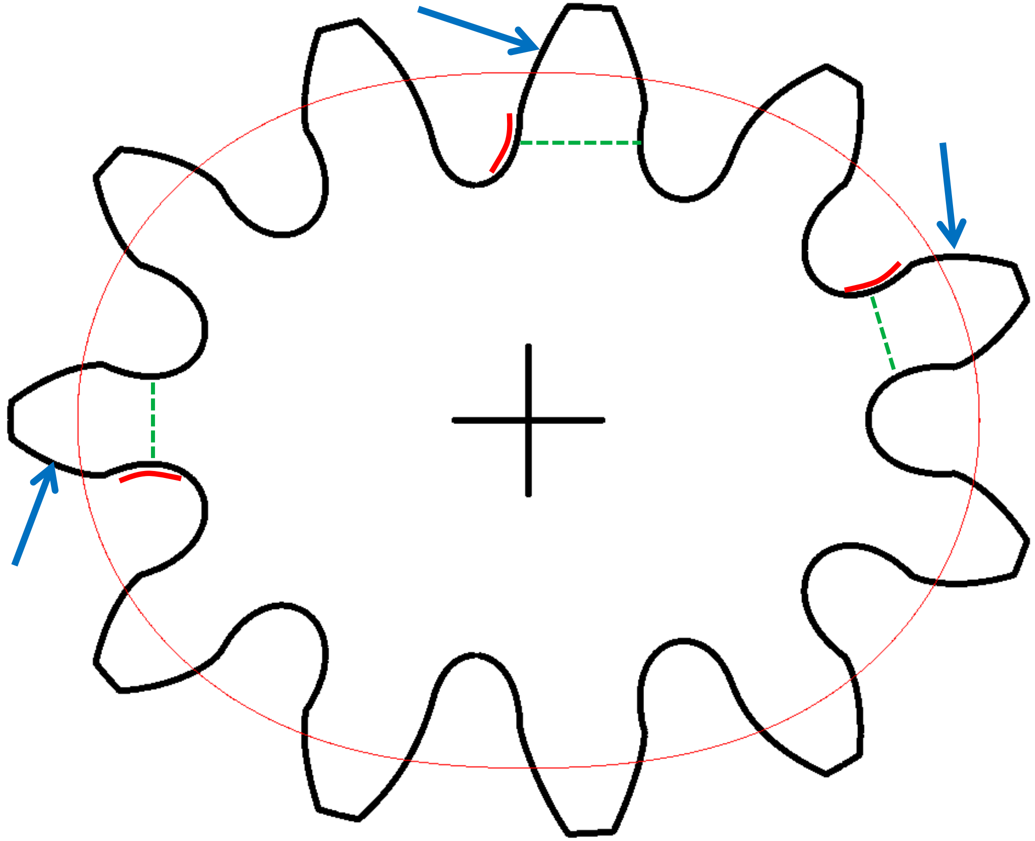
When using sintered or plastic material, then hobbing cutters or pinion type cutters are not required.





Strength calculation

- No standardized strength calculation is available for noncircular gears
- Noncircular gear can be momentarily represented as a cylindrical gear
- Calculation with replacement cylindrical gear can be used
- Identification of critical teeth
  - Most loaded tooth
  - Tooth with the highest undercut
  - Ratio load/root thickness
  - Smallest root radius
  - ...



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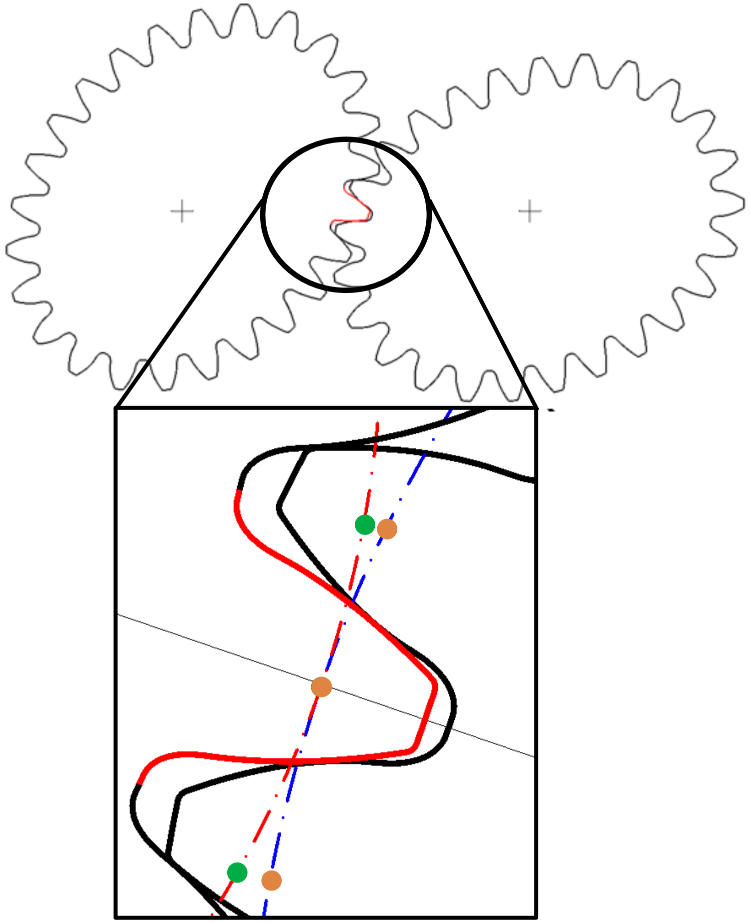
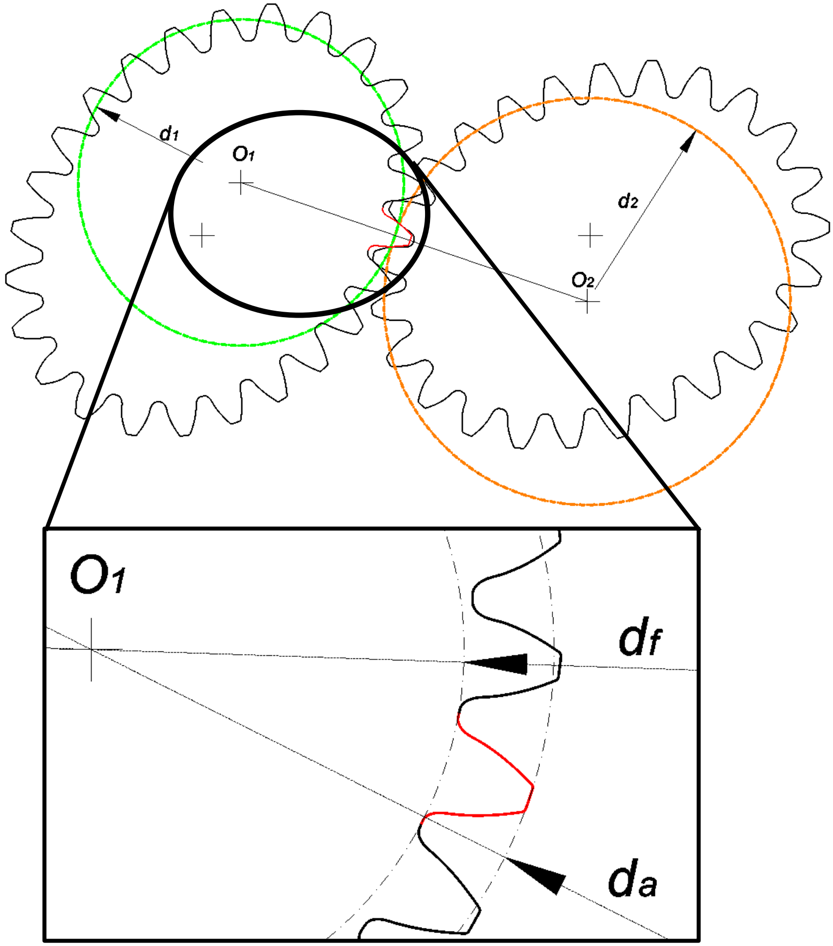
Strength calculation

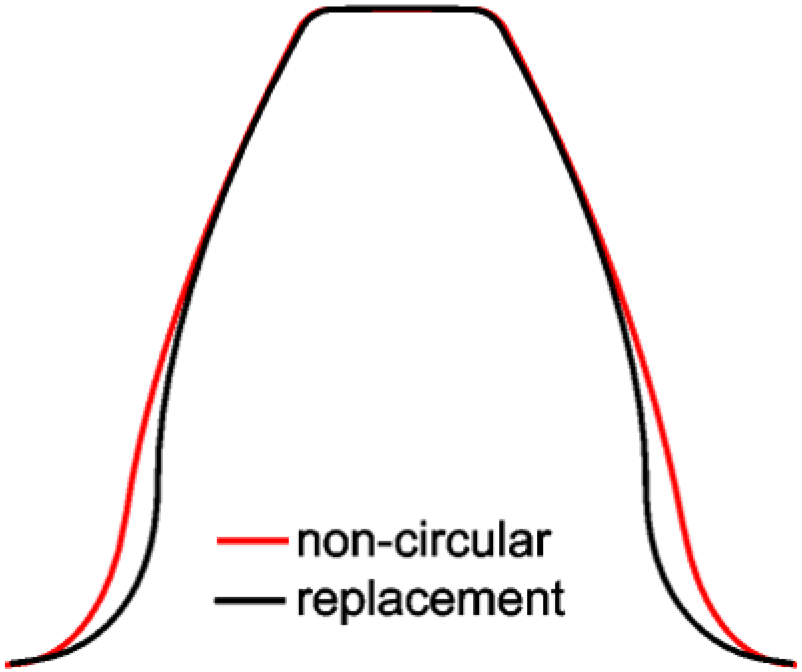
- Creating cylindrical replacement gear
- Operating pitch diameters from 'local' curvatures





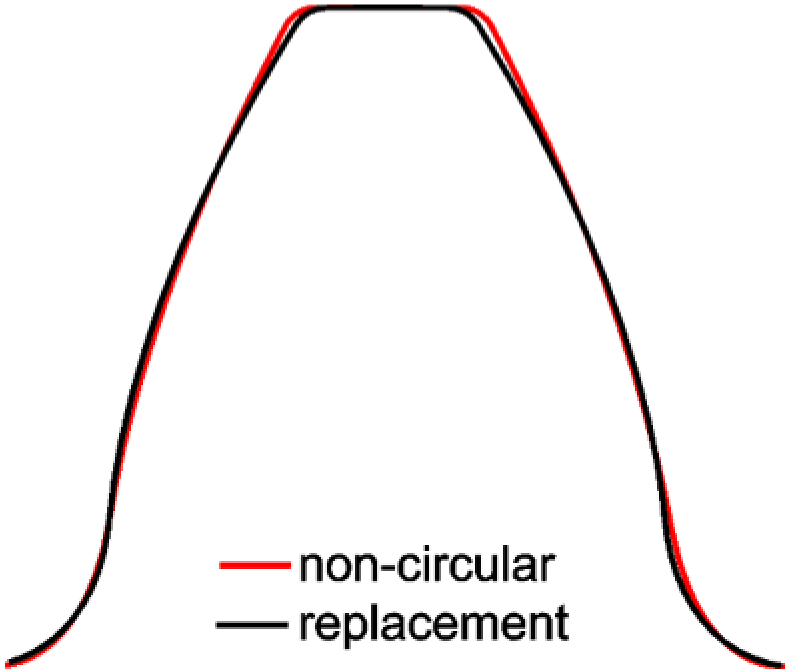
Strength calculation

- Normal module and pressure angle from noncircular gears
- Tip/root diameters for both gears
- Centre distance



m_n = 1.6123 mm, α = 20°

ρ_{fp} = 0.61 mm, A_{SN} = -0.05 mm



m_n = 1.6123 mm, α = 23°

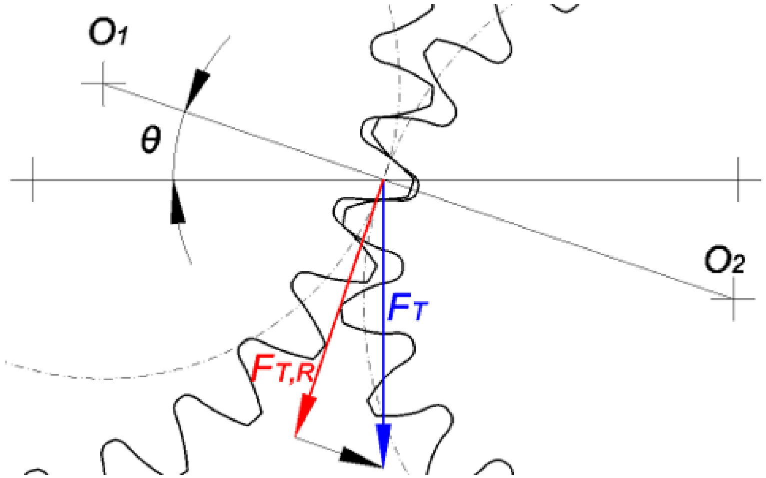
ρ_{fp} = 0.90 mm, A_{SN} = 0.00 mm

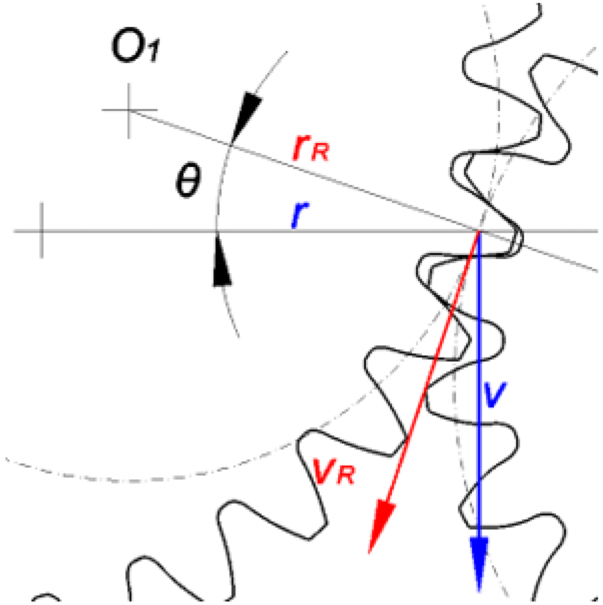
Strength calculation

- Tangential force and rotational speed have to be recalculated based on the new center points O_1 and O_2
- Strength calculation is then performed using ISO 6336 or VDI 2736

$$F_{T,R} = F_T / cos\theta$$

$$n_R = \frac{r \cdot n \cdot cos\theta}{r_R}$$





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Noncircular gears in KISSsoft

- The operating pitch lines are currently calculated in an Excel (and imported in KISSsoft)

NCG elliptical pitch line gear calculation (G1-G2)

Scale geometry

	k (intiger)	PK	EK
Ellipse with:	1	8.000000	0.500000

Scale factor	10
Ratio	1
Speed Gear 1 [rpm]	1
Center distance CD [mm]	21.333
Start angle [°]	0
End angle [°]	360
Angle increment [°]	0.1
tproU= (sec)	60
First cell	18
Last cell	3618

Operating pitch line length (mm)	
G1	62.61171
G2	62.61171

Rotation angle (°)	
G1	360.0000
G2	360.0000

SET eccentricity	
focal point = 1	
center point = 2	
user input = 3	
Offset [mm]	

MAX ratio	MIN ratio
3.0000	0.3333

SOLVER
5.8253E-20

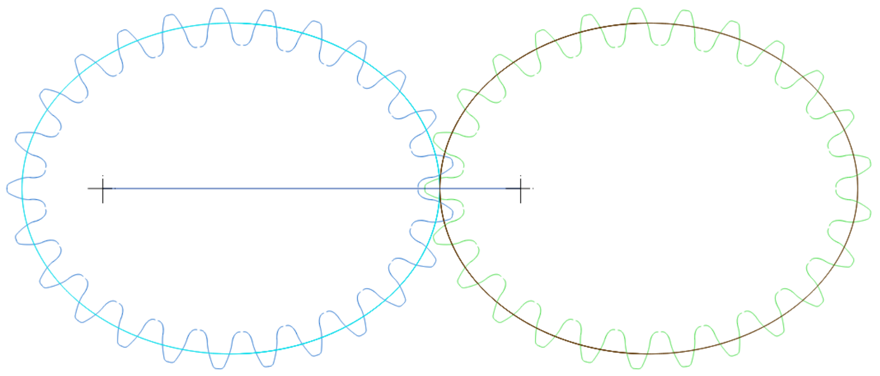
Solve CD | Solve PK | Solve EK

Calculate module G1	
z	25
mn	0.797197

Calculate number of teeth	
mn	0.797197
z1	25.000
z2	25.000

```
#Radius #Angle
15.44860651 -50
15.4695275  -49.9
15.49047438 -49.8
15.51144708 -49.7
15.53244549 -49.6
15.55346954 -49.5
15.57451913 -49.4
15.59559419 -49.3
15.61669461 -49.2
15.63782032 -49.1
15.65897121 -49
```

- The operating pitch lines are defined in polar coordinates or as a *.dxf file
- The operating pitch lines must be prolonged at the beginning and end for at least 30º



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Noncircular gears in KISSsoft

- Several different options available in KISSsoft

Center distance: FIXED

Input: OPERATING PITCH LINE

Type of center distance		fixed	
Center distance	a	30.00000000	mm

Generate

Specification	Operating pitch line	
Operating pitch line gear 1	ellipse_G1.DAT	...

Input: TRANSMISSION RATIO

Type of center distance		fixed	
Center distance	a	30.00000000	mm

Generate

Specification	Ratio	
Ratio	Input...	...

Center distance: VARIABLE

Input: OPERATING PITCH LINE

Type of center distance	variable

Generate

Specification	Operating pitch line
Operating pitch line gear 1	ellipse_G1.DAT
Operating pitch line gear 2	Input...

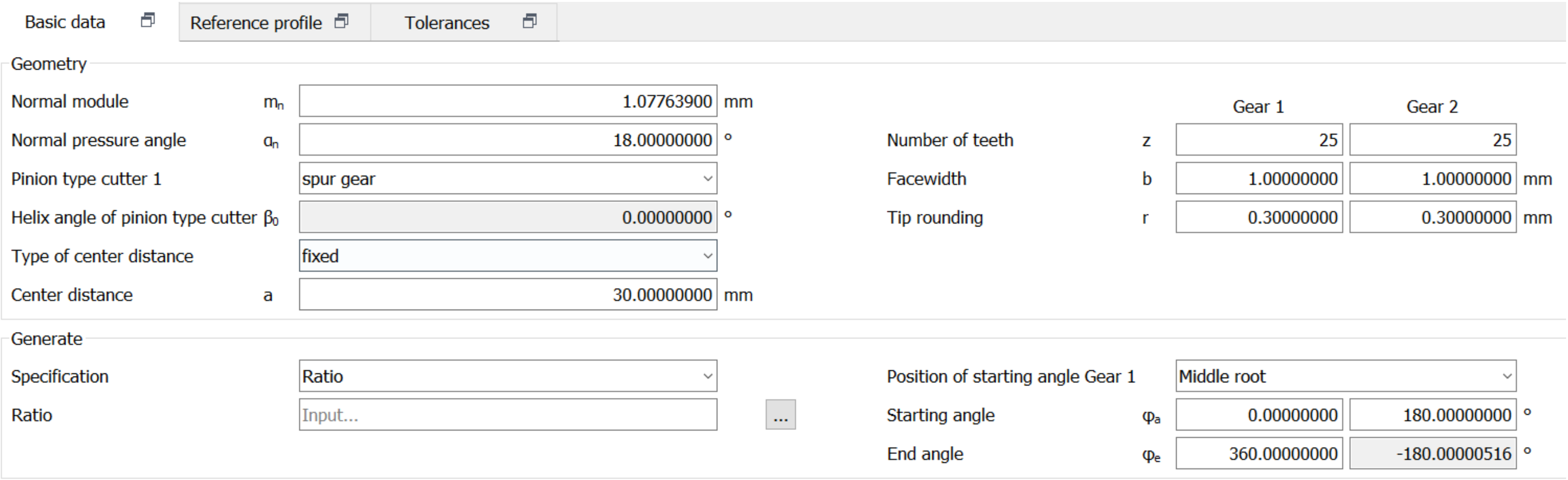
Input: TRANSMISSION RATIO

Type of center distance	variable

Generate

Specification	Ratio
Ratio	Input...
Operating pitch line gear 1	ellipse_G1.DAT

Noncircular gears in KISSsoft – Tab Basic data



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Noncircular gears in KISSsoft – Tab Reference profile

Basic data | Reference profile | Tolerances

Final machining Gear 1

Tool selection		Cutter/Tool: Pinion type cutter	
Designation		Own Input	
Number of teeth	z_0	20	
Profile shift coefficient	x_0	0.00000000	
Addendum coefficient	h^*_{aP0}	1.45000000	
Tip form		Rounding	
Tip radius coefficient	ρ^*_{aP0}	0.05000000	
Dedendum coefficient	h^*_{fP0}	1.00000000	
Root radius coefficient	ρ^*_{fP0}	0.05000000	
Protuberance height coefficient	h^*_{prP0}	0.00000000	
Protuberance angle	α_{prP0}	0.00000000	°
Root form height coefficient	h^*_{FfP0}	0.00000000	
Profile angle of the chamfer flank	α_{KP0}	0.00000000	°

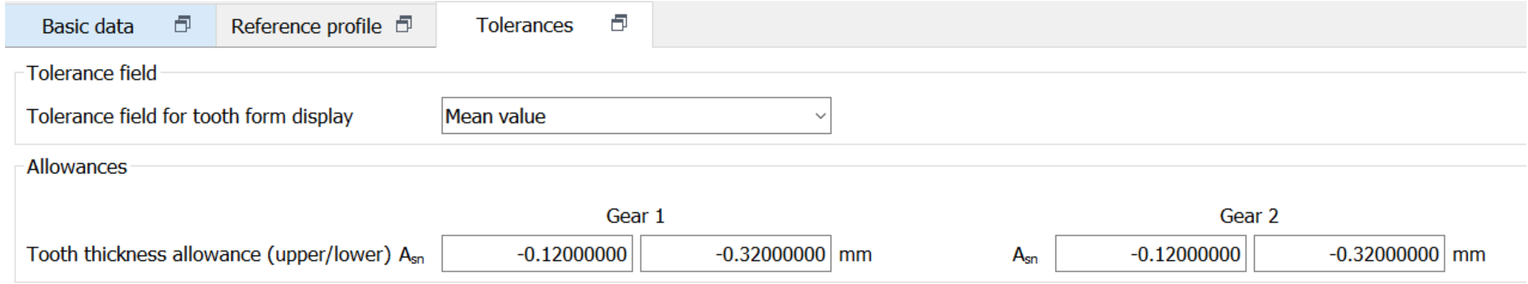
☑ Topping tool

Final machining Gear 2

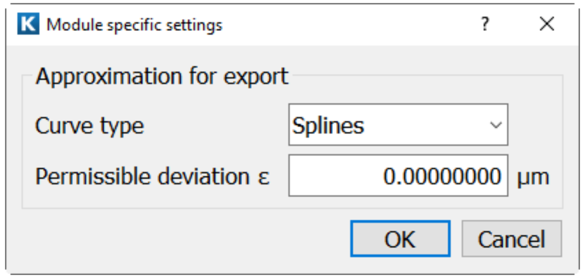
Tool selection		Cutter/Tool: Pinion type cutter	
Designation		Own Input	
Number of teeth	z_0	20	
Profile shift coefficient	x_0	0.00000000	
Addendum coefficient	h^*_{aP0}	1.45000000	
Tip form		Rounding	
Tip radius coefficient	ρ^*_{aP0}	0.05000000	
Dedendum coefficient	h^*_{fP0}	1.00000000	
Root radius coefficient	ρ^*_{fP0}	0.05000000	
Protuberance height coefficient	h^*_{prP0}	0.00000000	
Protuberance angle	α_{prP0}	0.00000000	°
Root form height coefficient	h^*_{FfP0}	0.00000000	
Profile angle of the chamfer flank	α_{KP0}	0.00000000	°

☑ Topping tool

Noncircular gears in KISSsoft – Tab Reference profile

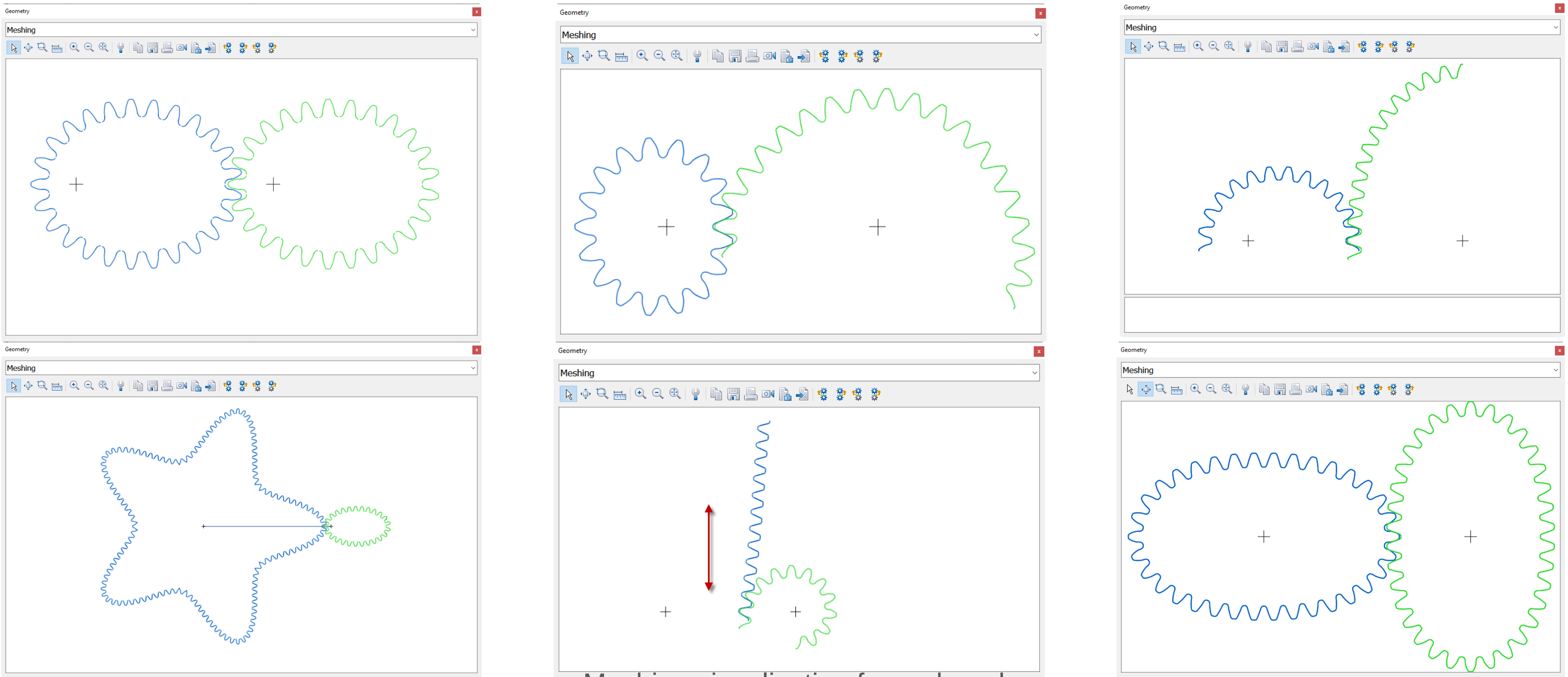


Module specific settings



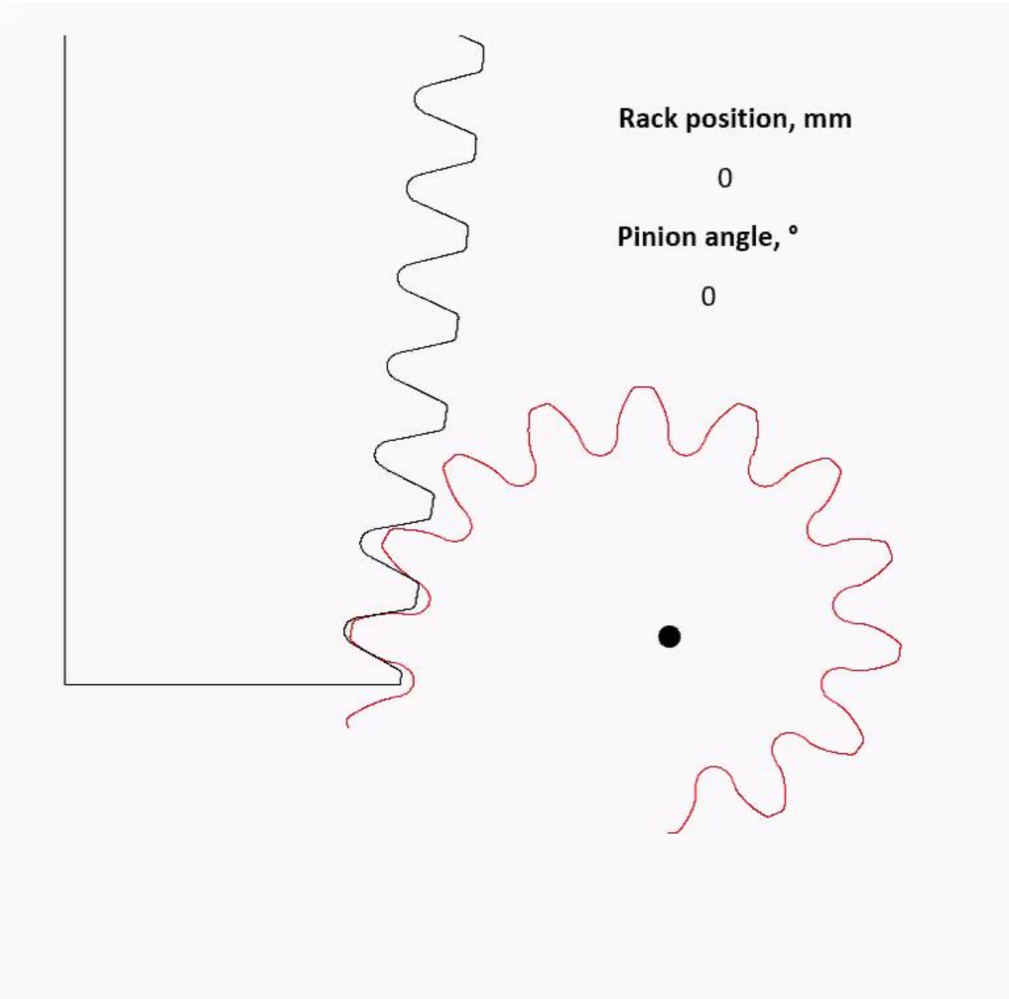
KISSsoft

Noncircular gears in KISSsoft – Tooth form examples



Meshing visualization for rack and pinion must be performed in Excel

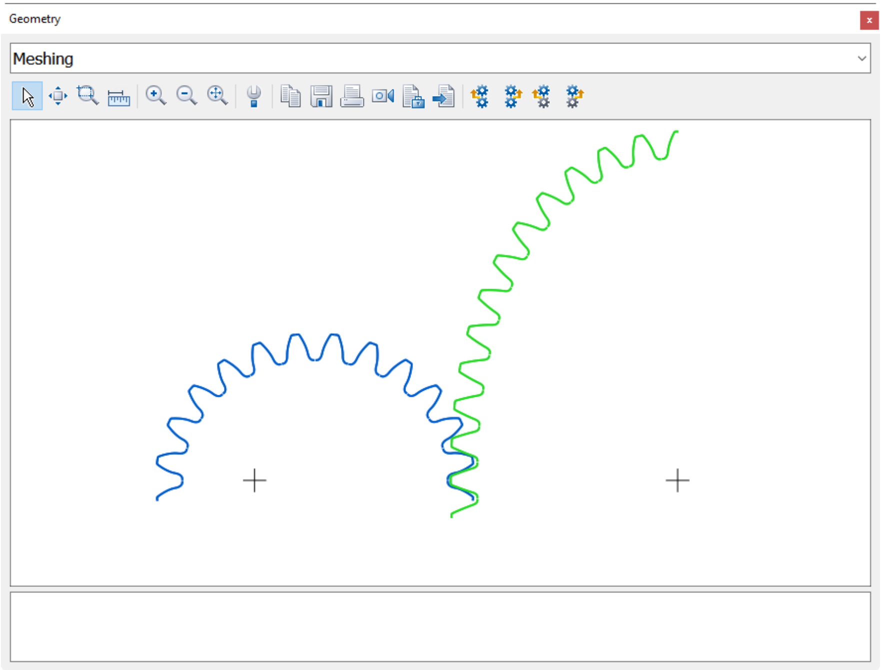
Noncircular pinion and rack



KISSsoft

Case study 1

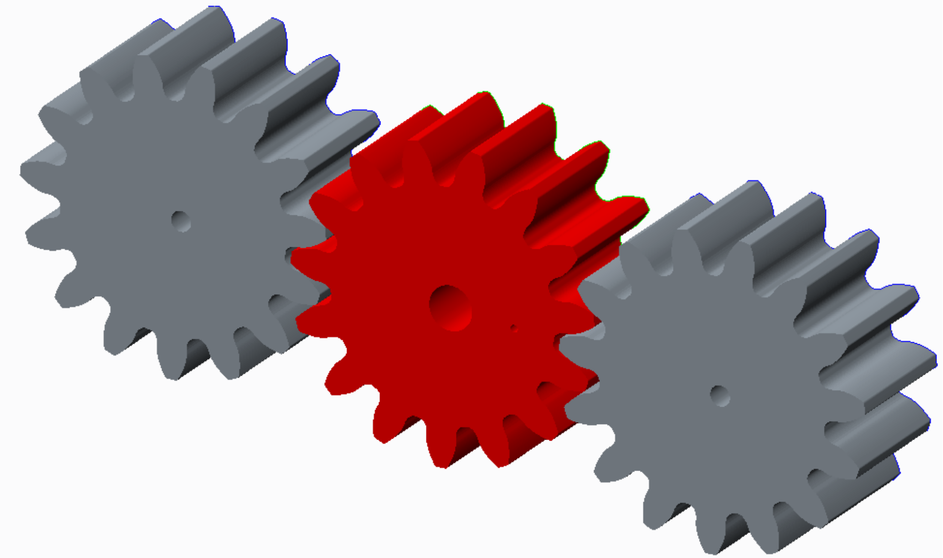
- Cylindrical gears with constant transmission ratio i = 10
- Implementation of the non-circular gear stage
  - Reducing the motor size to half
  - Implementation of the stop positions in the design



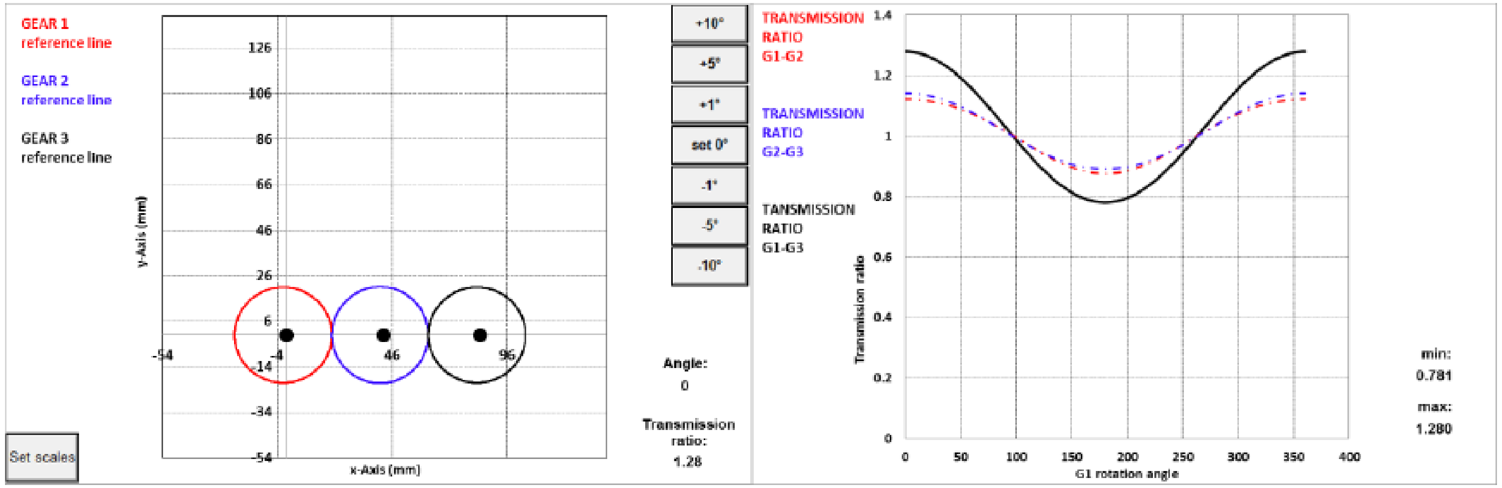
KISSsoft

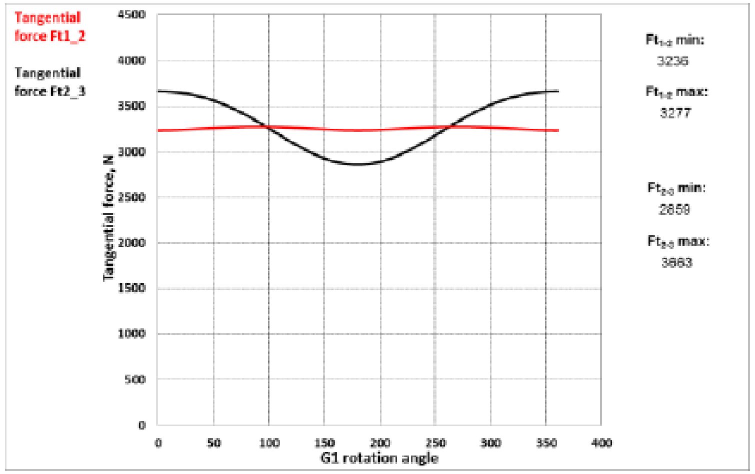
Case study 2

- 3 non-circular gear train with transmission ratio i = 1 ± 0.25



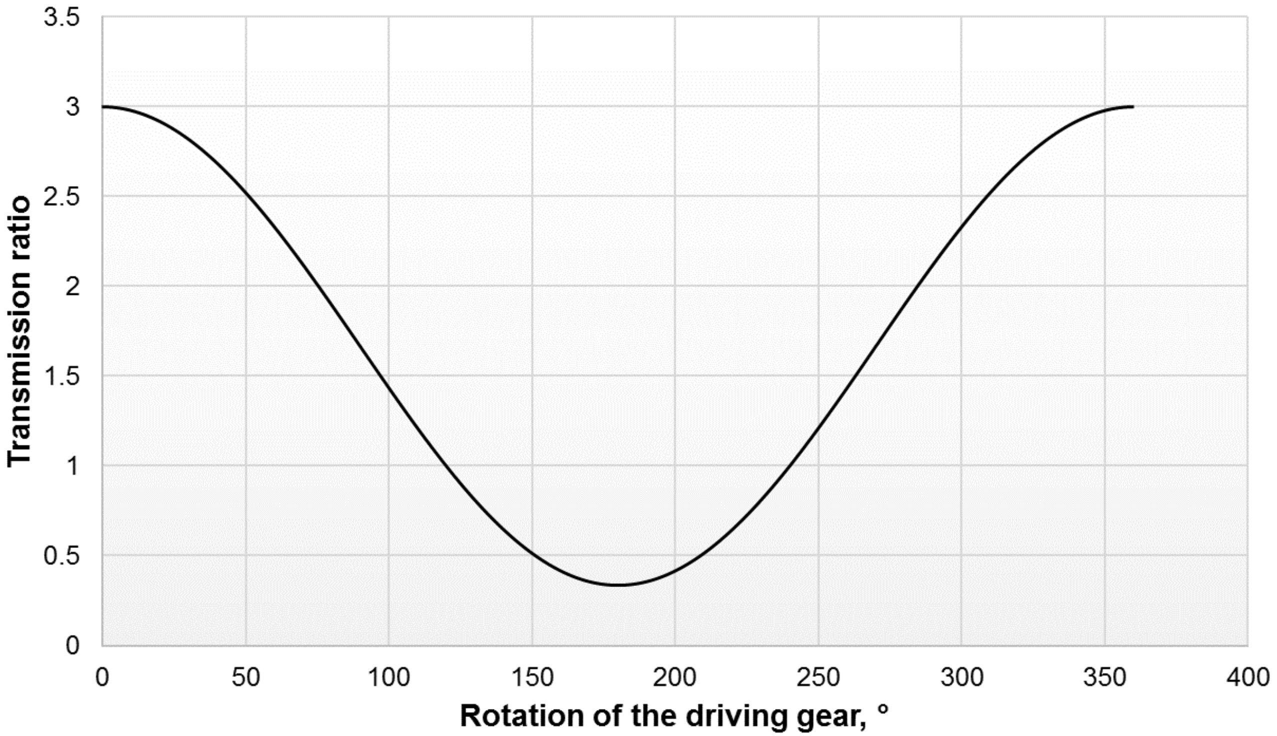
1st and 3rd gears are cylindrical gears with an offset of 0.65 mm





Case study 3

- Cable braided sleeving machines
- Gears with revolutions 1:1
- Variation of transmission ratio: 1:6



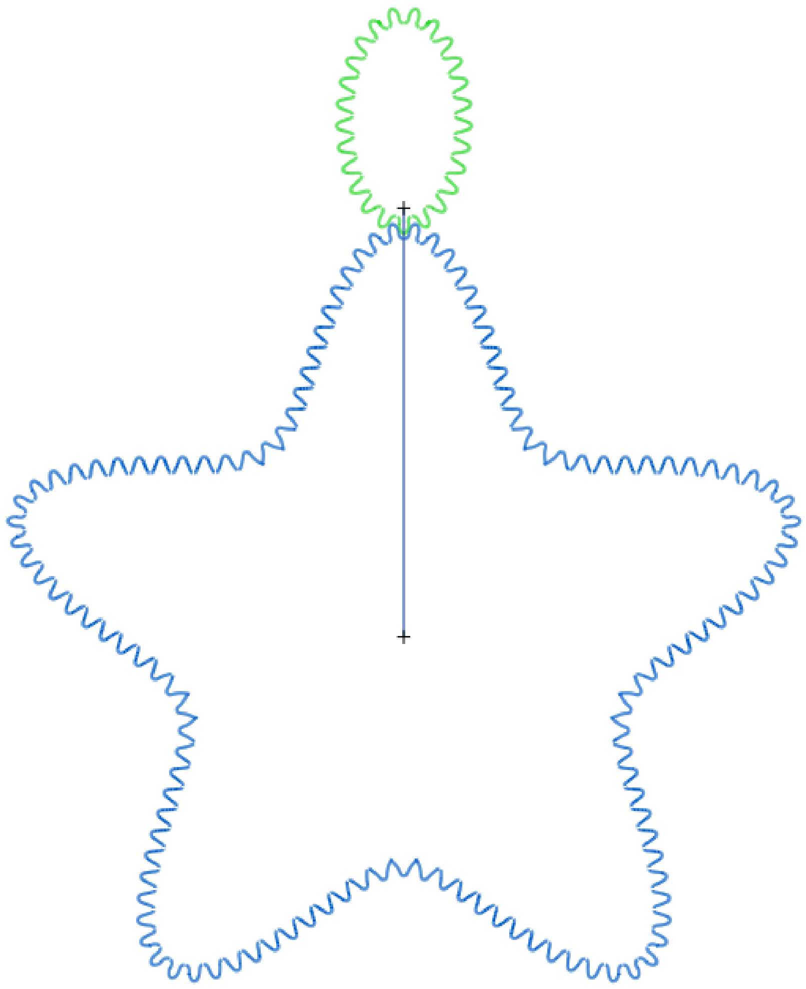
KISSsoft

Module Z40

- Currently the KISSsoft module is sold together with a 1-time engineering
- The customer gets the Excel file to generate the operating pitch lines and instructions on how to generate the tooth form in KISSsoft
- He can then modify the Excel and use it for his next projects

KISSsoft

Thank you for your attention!



Sharing Knowledge

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