Derivation of Tooth Stiffness of Asymmetric Gears for Loaded Tooth Contact Analysis

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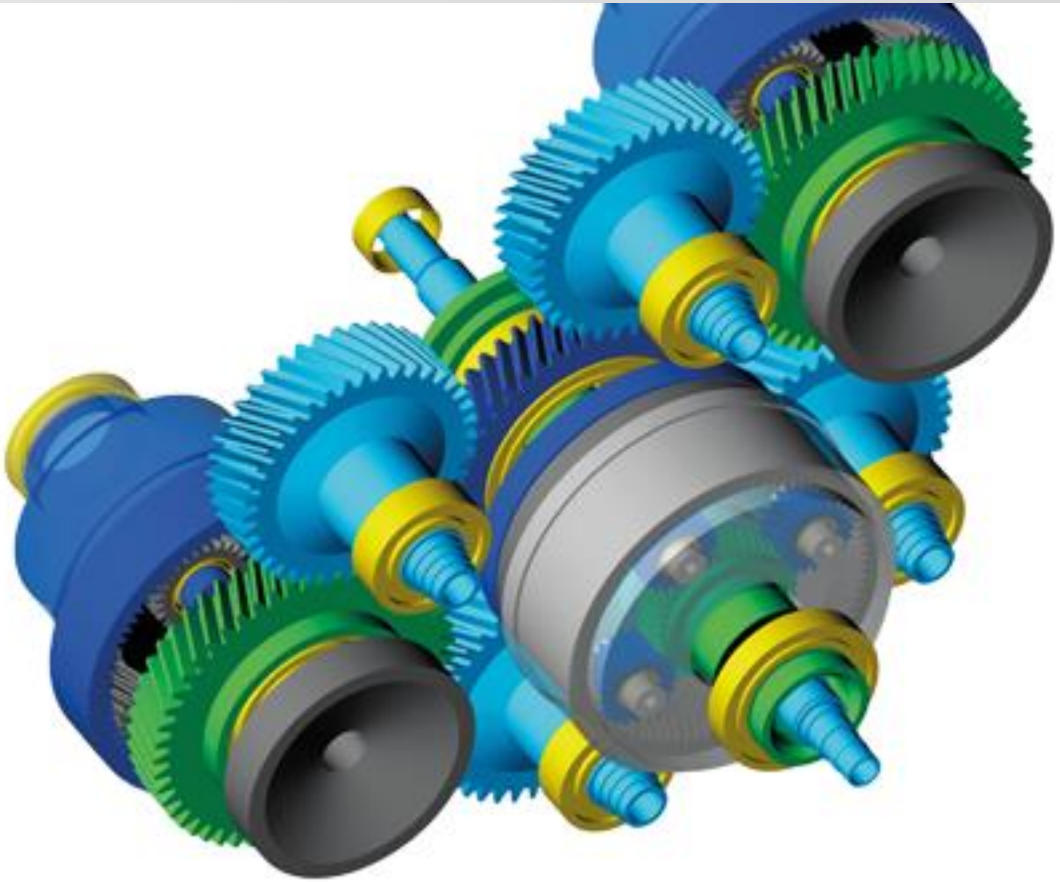

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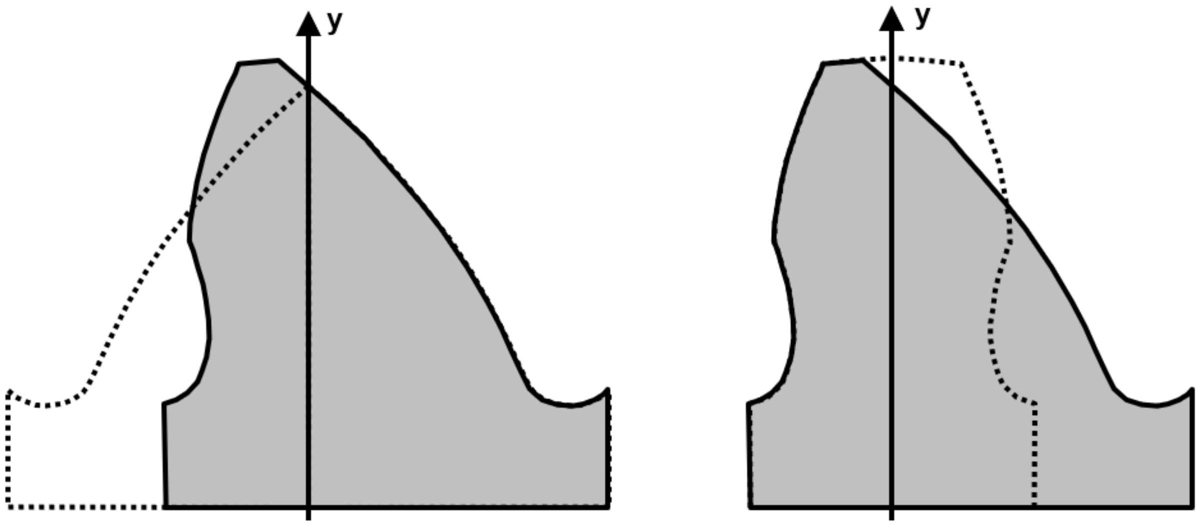
What are asymmetric gears?

- Asymmetric gears stand out by an individual pressure angle of each flank
- Can have significantly larger pressure angle compared to symmetric gears without violating minimum topland criteria



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Why can asymmetric teeth be of interest?

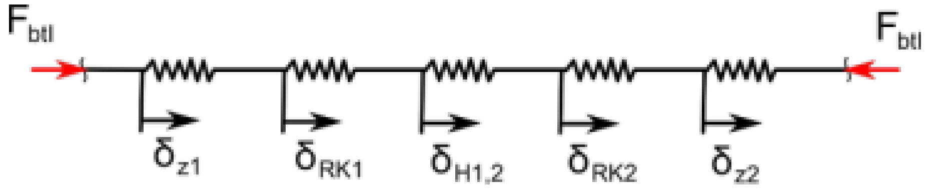
- Increased torque capacity for gears with mostly unidirectional load
- Reduced contact stress due to high pressure angle
- Reduced power loss

Drawback:

- Low contact ratio and therefore asymmetric gears can produce more noise
- More expensive to produce (in case of steel gears)

Tooth Stiffness of Symmetric Teeth

1. Stiffness calculation based upon deflection calculation according to the paper "Formänderung und Profilrücknahme bei gerad- und schrägverzahnten Rädern" (Weber/Banaschek)
2. Weber/Banaschek defines 3 components
   - Bending $\delta_b \rightarrow C_b = \frac{1}{\delta_b}$
   - Tilting $\delta_t \rightarrow C_t = \frac{1}{\delta_t}$
   - Hertzian Flattening $\delta_{H1,2} \rightarrow C_{H1,2} = \frac{F}{\delta_{H1,2}}$
3. $\delta = \delta_{b1} + \delta_{b2} + \delta_{t1} + \delta_{t2} + \delta_{H1,2}$
4. $\frac{1}{C} = \frac{1}{C_{b1}} + \frac{1}{C_{b2}} + \frac{1}{C_{t1}} + \frac{1}{C_{t2}} + \frac{1}{C_{H1,2}}$



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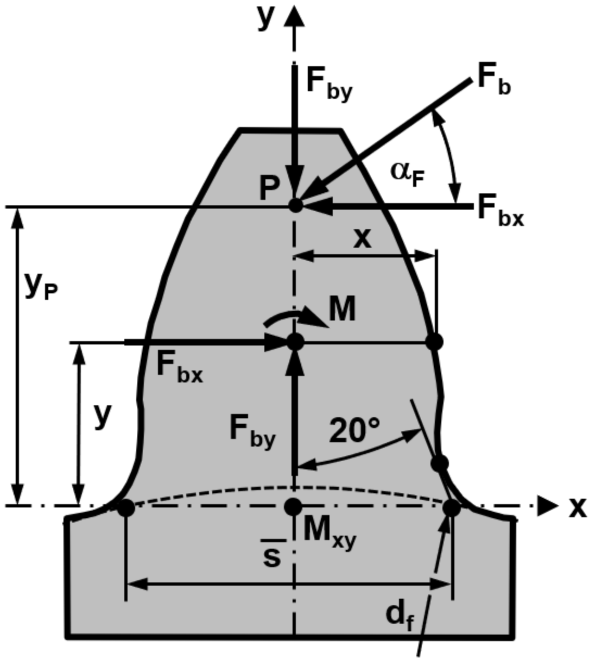
Tooth Stiffness of Symmetric Teeth (tilting and bending)



$$\frac{1}{2}P\delta_b = \frac{1}{2}\int_0^{y_p} \frac{M^2}{\frac{E}{1-\nu^2}\cdot\frac{b}{12}(2x)^3}\mathrm{d}y + \frac{1}{2}\int_0^{y_p}\frac{1.2F_{bx}^2}{G\ b\ 2x}\mathrm{d}y + \frac{1}{2}\int_0^{y_p}\frac{F_{by}^2}{\frac{E}{1-\nu^2}b\ 2x}\mathrm{d}y$$

$$\frac{1}{2}P\delta_t = \frac{9(1-\nu^2)}{\pi E b\bar{s}^2}M^2 + 2\frac{(1+\nu)(1-2\nu)}{2Eb\bar{s}}MF_{bx} + \frac{2.4(1-\nu^2)}{\pi Eb}F_{bx}^2 + 0.294\tan\alpha_F\frac{2.4(1-\nu^2)}{\pi Eb}F_{by}^2$$

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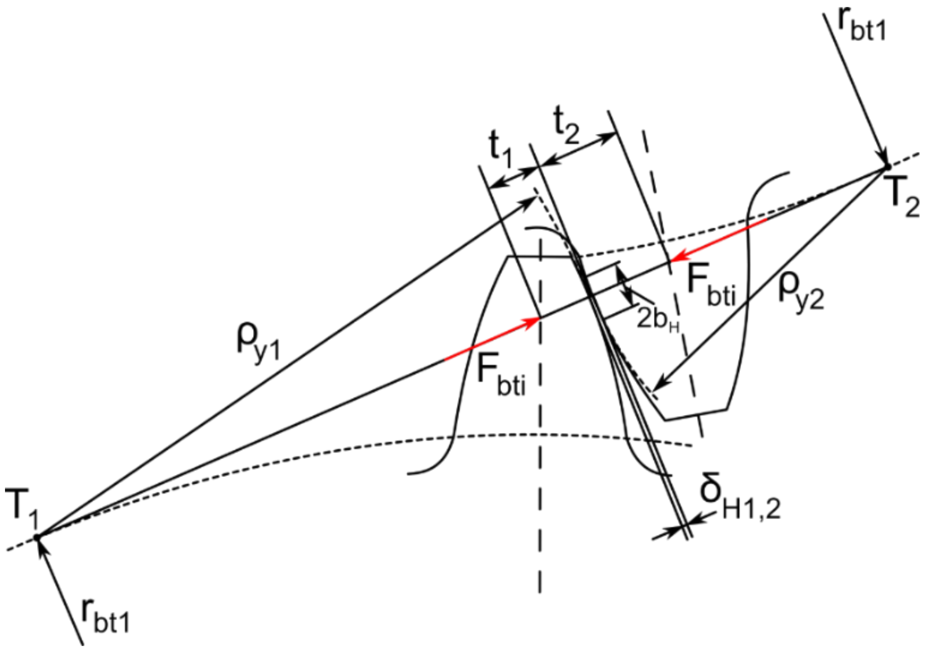
Tooth Stiffness of Symmetric Teeth (Hertzian flattening)



$$\delta_{H1,2} = \frac{F_{bti}}{\pi b_g}\left[\left|\frac{1-\nu_1^2}{E_1}\ln\left(\frac{b_H^2}{4t_1^2}\right)+\frac{\nu_1(1+\nu_1)}{E_1}\right|+\left|\frac{1-\nu_2^2}{E_2}\ln\left(\frac{b_H^2}{4t_2^2}\right)+\frac{\nu_2(1+\nu_2)}{E_2}\right|\right]$$

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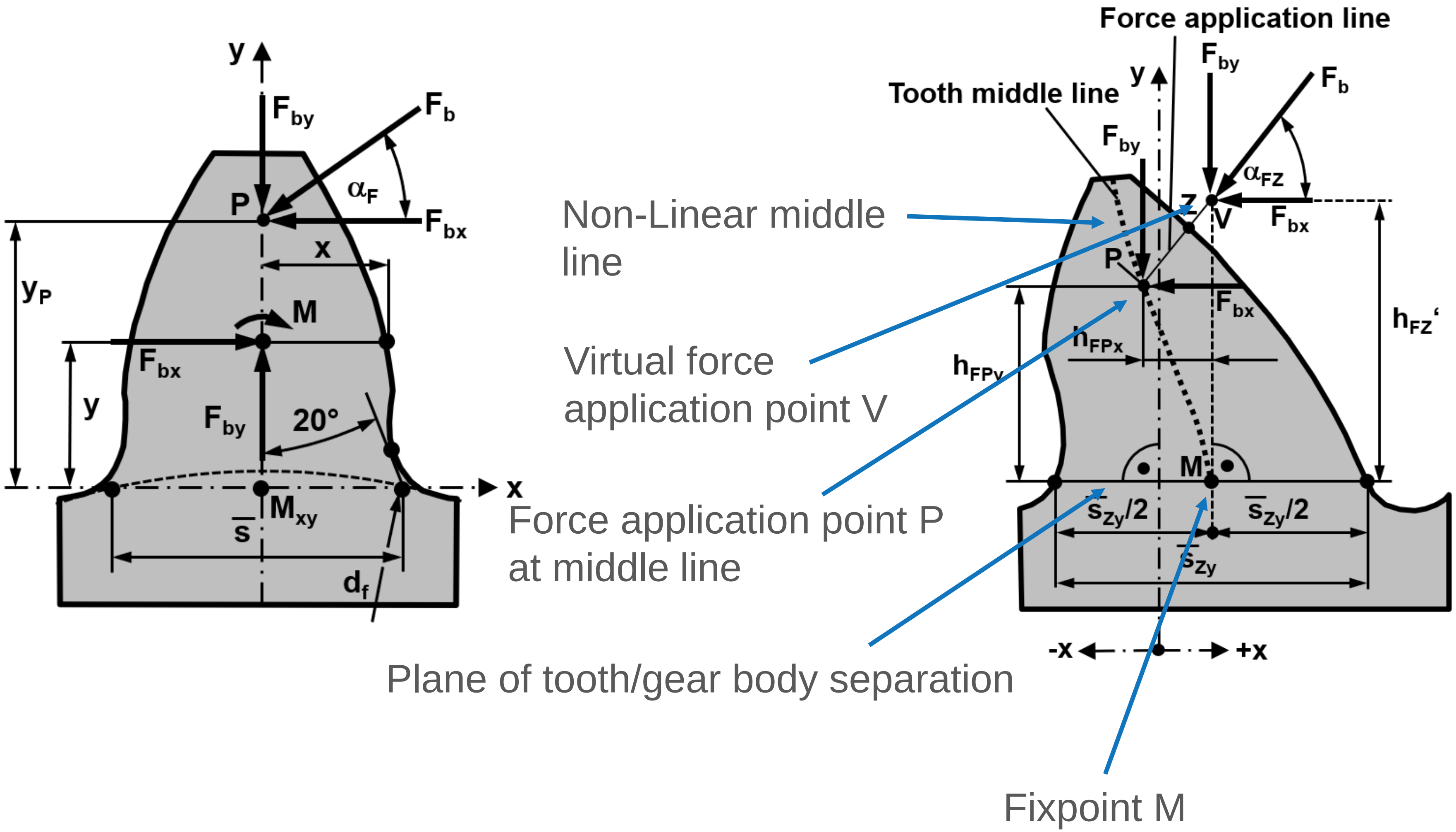
Tooth Stiffness of Asymmetric Teeth according to Dissertation of Andreas Langheinrich: "Geometrie, Beanspruchung und Verformung asymmetrischer Stirnradverzahnungen", 2014, Technische Universität München

Tooth Stiffness of Asymmetric Teeth



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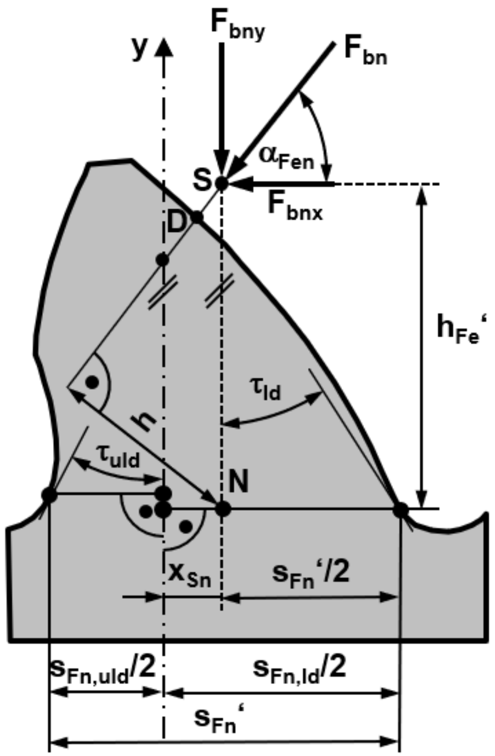
The plane of tooth and gear body separation can't be defined by the intersection of the 20° root fillet tangent with the root diameter d_f



Tooth Stiffness of Asymmetric Teeth

Bending: $\frac{1}{2}P\delta_b = \frac{1}{2}\int_0^{h_{FPy}} \frac{M^2}{\frac{E}{1-\nu^2}\cdot\frac{b}{12}\bar{S}_{Zy}^3}\,\mathrm{d}y + \frac{1}{2}\int_0^{h_{FPy}} \frac{1.2F_{bx}^2}{G\;b\;\bar{S}_{Zy}}\,\mathrm{d}y + \frac{1}{2}\int_0^{h_{FPy}} \frac{F_{by}^2}{\frac{E}{1-\nu^2}b\;\bar{S}_{Zy}}\,\mathrm{d}y$

Tilting: $\frac{1}{2}P\delta_t = \frac{9\left(1-\nu^2\right)}{\pi E b \bar{S}_{Zy}^2}M^2 + 2\frac{(1+\nu)(1-2\nu)}{2Eb\bar{S}_{Zy}}MF_{bx} + \frac{2.4\left(1-\nu^2\right)}{\pi E b}F_{bx}^2 + 0.294\tan\alpha_F\frac{2.4\left(1-\nu^2\right)}{\pi E b}F_{by}^2$

Flattening: $\delta_{H1,2} = \frac{F_{bti}}{\pi b_g}\left[\left|\frac{1-\nu_1^2}{E_1}\ln\left(\frac{b_H^2}{4t_1^2}\right) + \frac{\nu_1(1+\nu_1)}{E_1}\right| + \left|\frac{1-\nu_2^2}{E_2}\ln\left(\frac{b_H^2}{4t_2^2}\right) + \frac{\nu_2(1+\nu_2)}{E_2}\right|\right]$

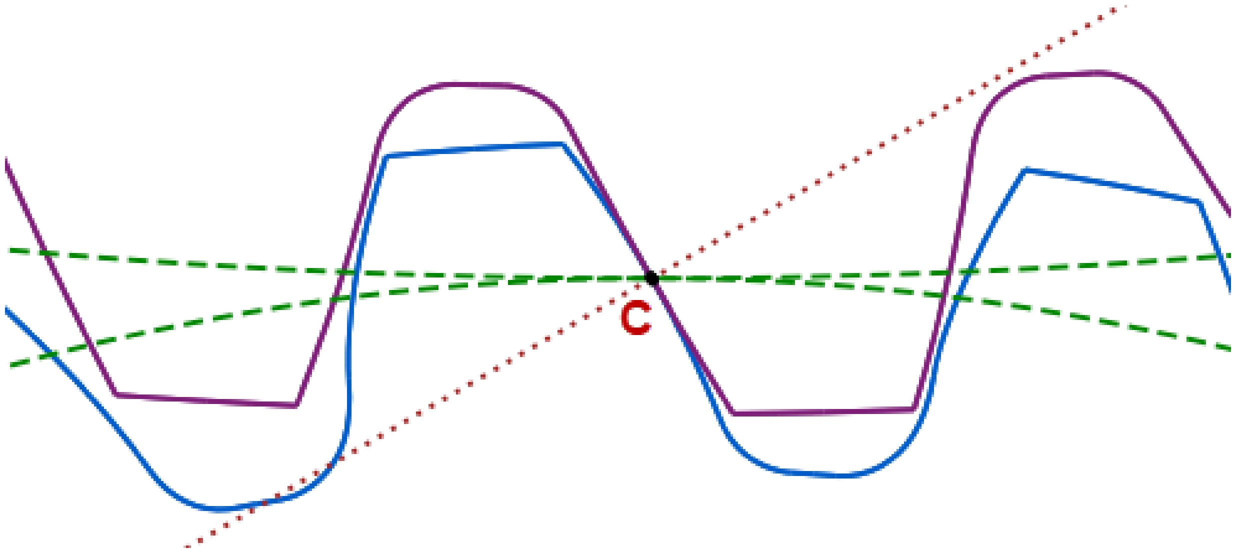
$$M = F_{bx}h'_{FZ} = F_{bx}(y_V - y_M)$$

$$y_V = y_Z + (x_M - x_Z)\tan\alpha_{FZ}$$

$$x_M = \frac{\bar{S}_{Zy}}{2}, y_M = \frac{d_f}{2}$$

Implications of Adapted Stiffness Calculation

α_{nL}/α_{nR}[°]	m_n [mm]	z_1/z_2	b [mm]	a [mm]
15/30	6	25/76	44	303
h_{fP}^*	ρ_{fP}^*	h_{aP}^*	F_n[N/mm]	ε_α
0.955	0.38	0.705	532	1.494/0.981



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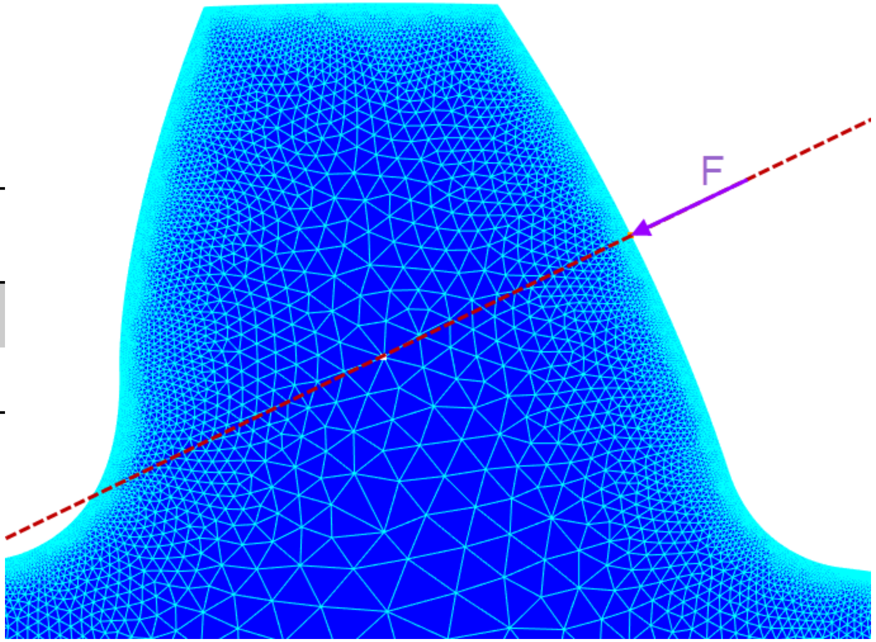
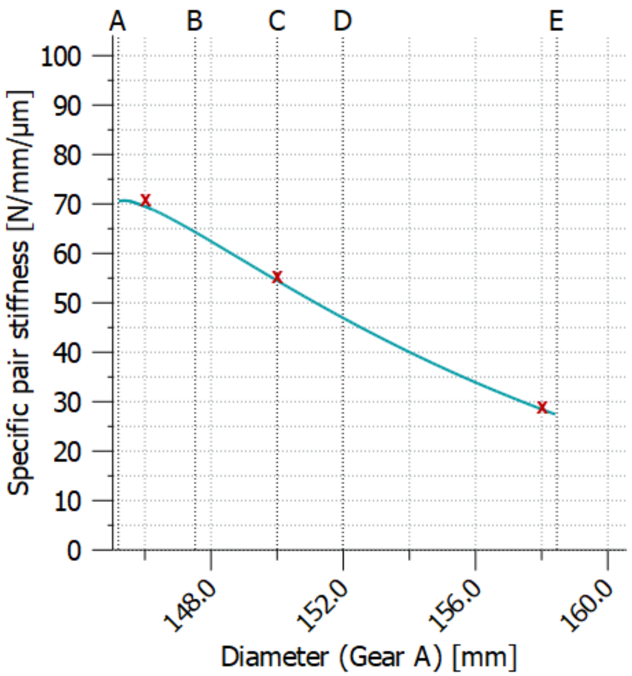
Implications of Adapted Stiffness Calculation

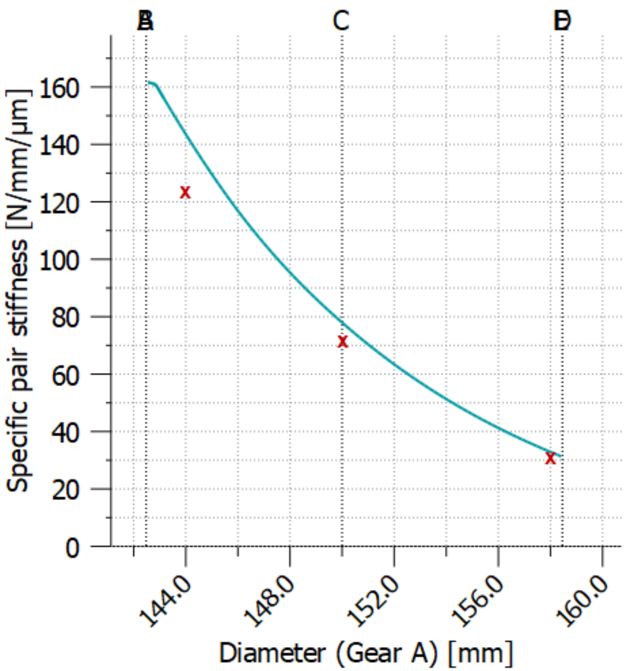
Type	$y_P = h_{FPy}$ [mm]	$M(x/y)$ [mm]	$\sigma_{F\ (Y_S Y_F)max}$ [N/mm²]	$\sigma_{F\ (Y_S Y_F)30°}$ [N/mm²]	FEMmax [N/mm²]	FEM30° [N/mm²]
Asymmetric R 30°	3.57	0.53/69.13	163.25	150.79	151.33	133.32
Asymmetric L 15°	4.76	0.53/69.13	206.23	192.02	214.63	203.09



Kissling U., Zotos I.: The Influence of a Grinding Notch on the Gear Bending Strength Rating, AGMA Technical Paper, 17FTM23, KISSsoft AG, 2017

Implications of Adapted Stiffness Calculation





--- = Asymmetric $\alpha_n = 15°$

X = Results of FEM calculation

--- = Asymmetric $\alpha_n = 30°$

X = Results of FEM calculation

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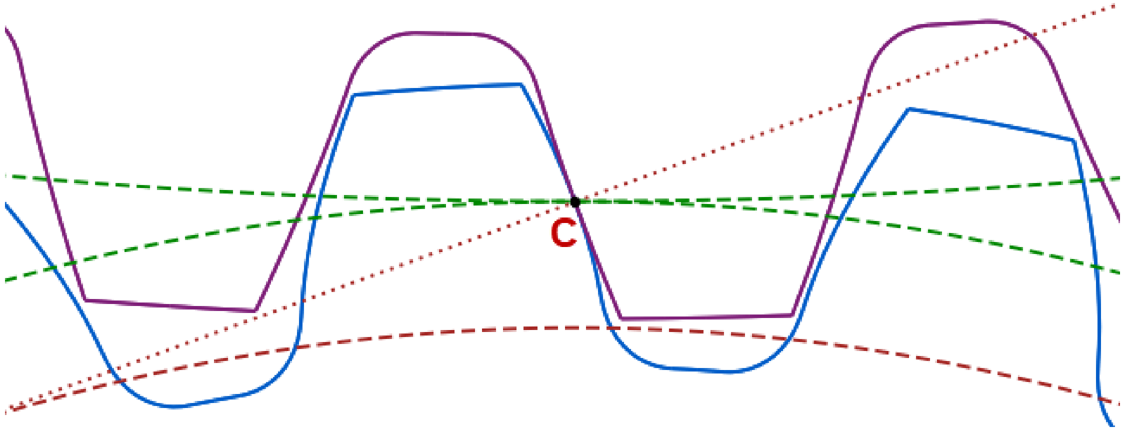
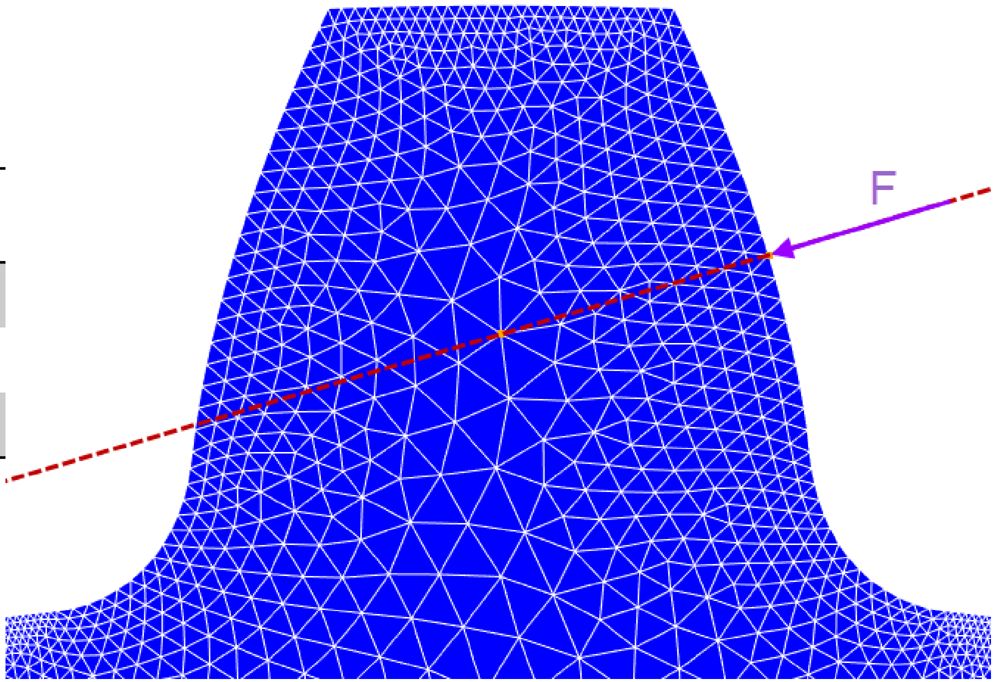
To prove Langheinrich's adaption of W/B, calculations of a symmetric and an almost symmetric tooth shape are compared.

α_{nL}/α_{nR}[°]	m_n [mm]	z_1/z_2	b [mm]	a [mm]
19.9999/20	6	25/76	44	303
h^*_{fP}	ρ^*_{fP}	h^*_{aP}	F_n[N/mm]	ε_α
0.955	0.38	0.705	532	1.245



Implications of Adapted Stiffness Calculation

Type	$y_P = h_{FPy}$ [mm]	$M(x/y)$ [mm]	$\sigma_{F\,(Y_sY_F)max}$ [N/mm²]	$\sigma_{F\,(Y_sY_F)30°}$ [N/mm²]	FEMmax [N/mm²]	FEM30° [N/mm²]
Symmetric	4.68	0/68.80	201.23	188.23	201.06	179.69
Asymmetric R	4.38	~0/69.11	201.22	188.22	201.31	179.97
Asymmetric L	4.38	~0/69.11	200.86	187.79	201.23	180.15



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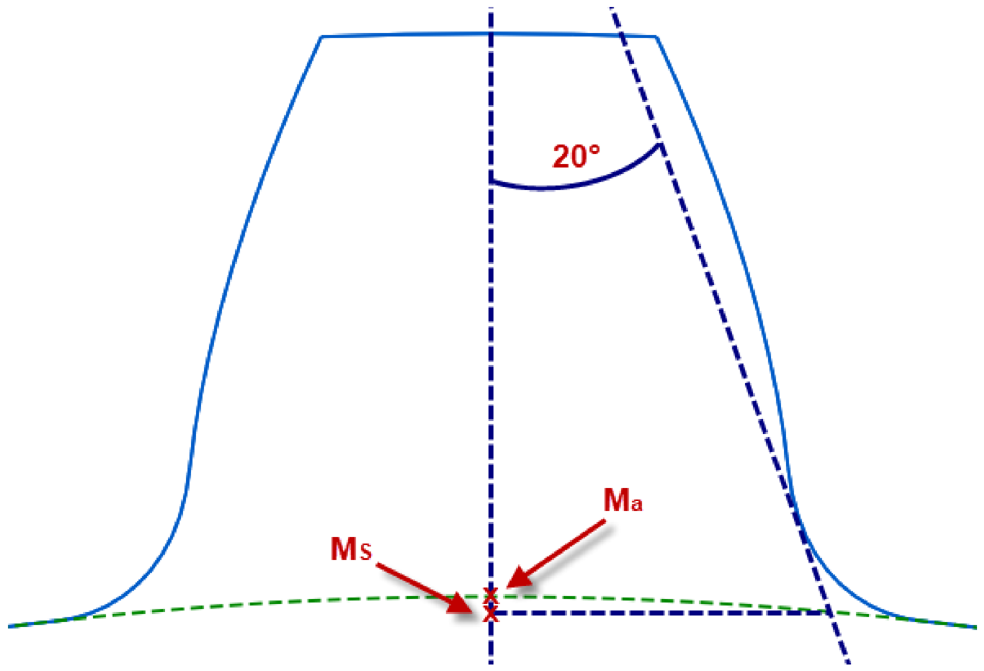
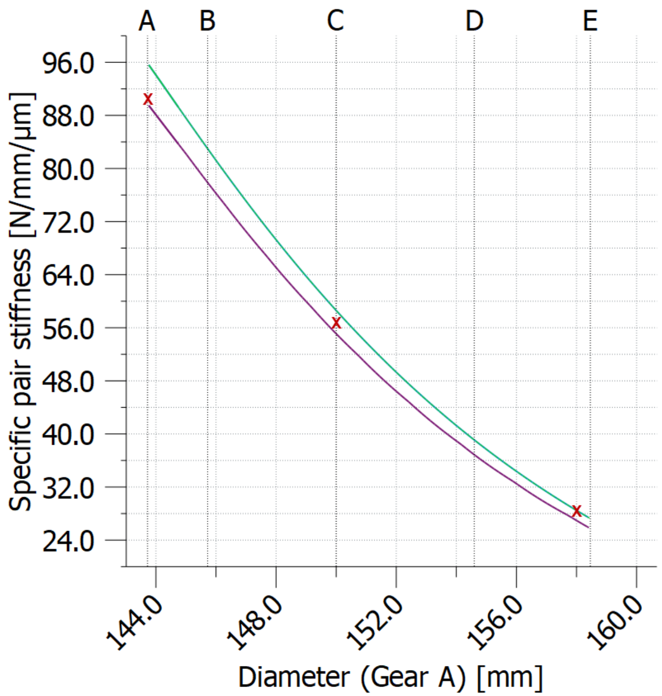
Implications of Adapted Stiffness Calculation



Type	$y_P = h_{FPy}$ [mm]	$M(x/y)$ [mm]
Symmetric	4.68	0/68.80
Asymmetric R	4.38	~0/69.11
Asymmetric L	4.38	~0/69.11

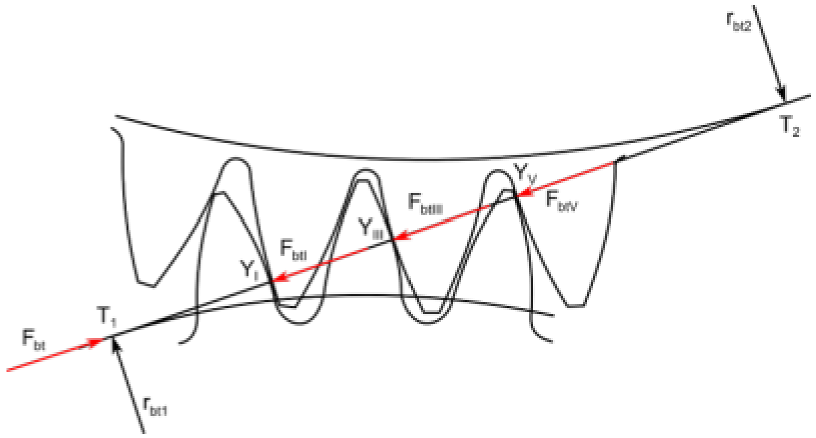


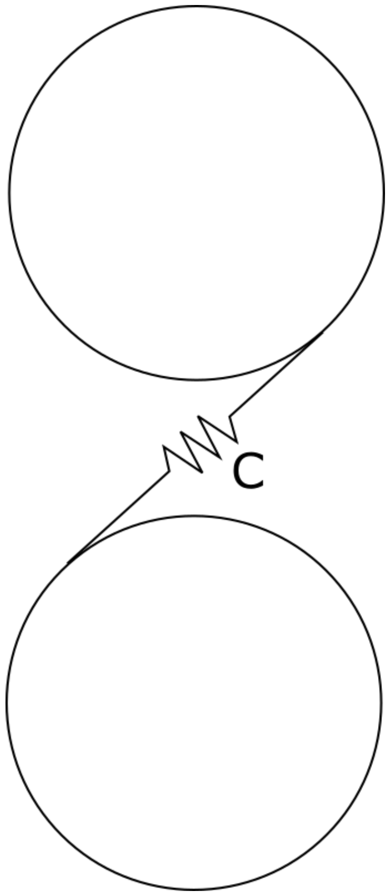
--- = Symmetric $\alpha_n = 20°$

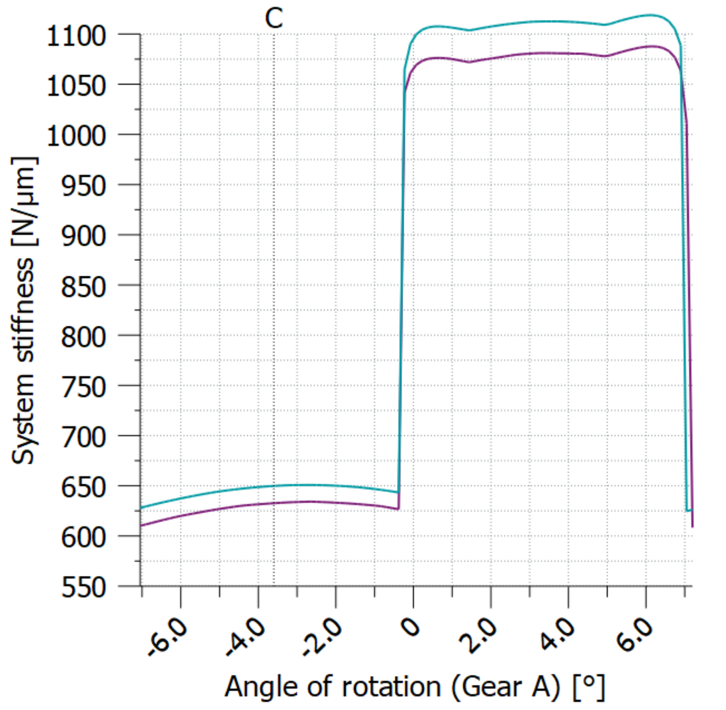
--- = Asymmetric $\alpha_n = 19.9999°/20°$

X = Results of FEM calculation

Implications of Adapted Stiffness Calculation







--- = Symmetric $\alpha_n = 20°$

--- = Asymmetric $\alpha_n = 19.9999°/20°$

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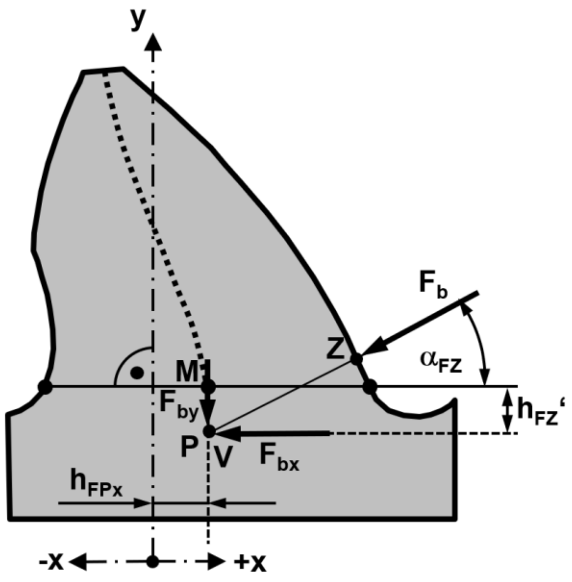
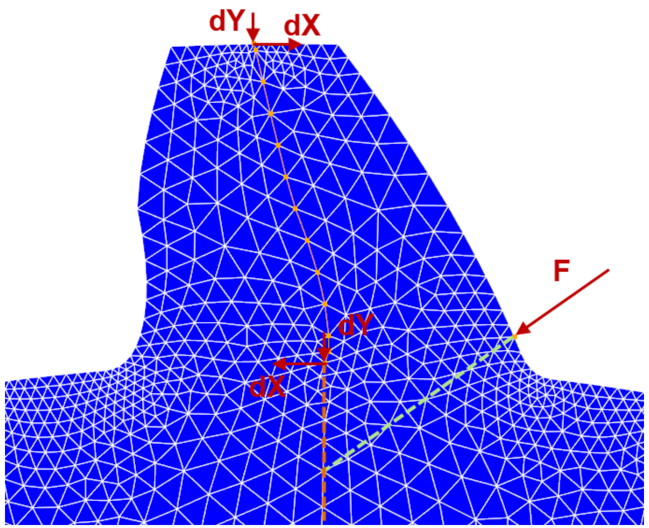
To compensate stiffness step effect, in case of $\alpha_{nL} \approx \alpha_{nR}$, two solutions are possible:

1. Considering fixpoint of W/B as $M_y = \frac{d_f}{2}$
2. Interpolate between W/B and $M_y = \frac{d_f}{2}$ approach

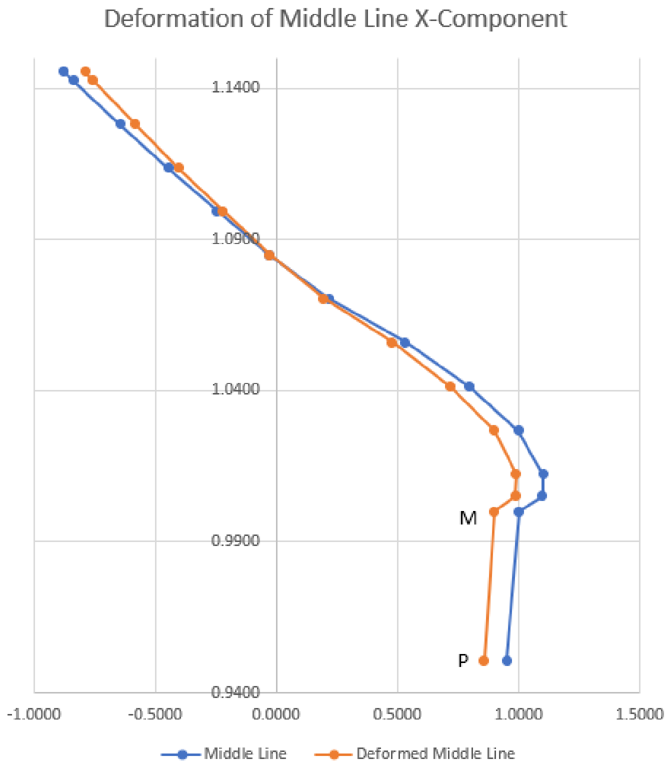
Approach according to W/B, no 20° root fillet tangent exist in case of high pressure angle

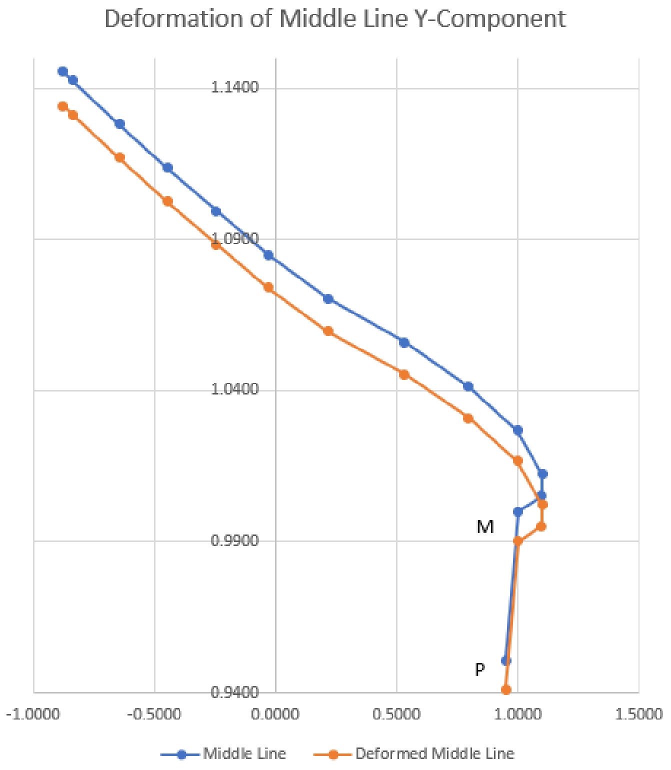




In case of a high pressure angle combined with the load close to the root fillet the lever h_{FZ}' of the load application point P gets negative.

Implications of Adapted Stiffness Calculation



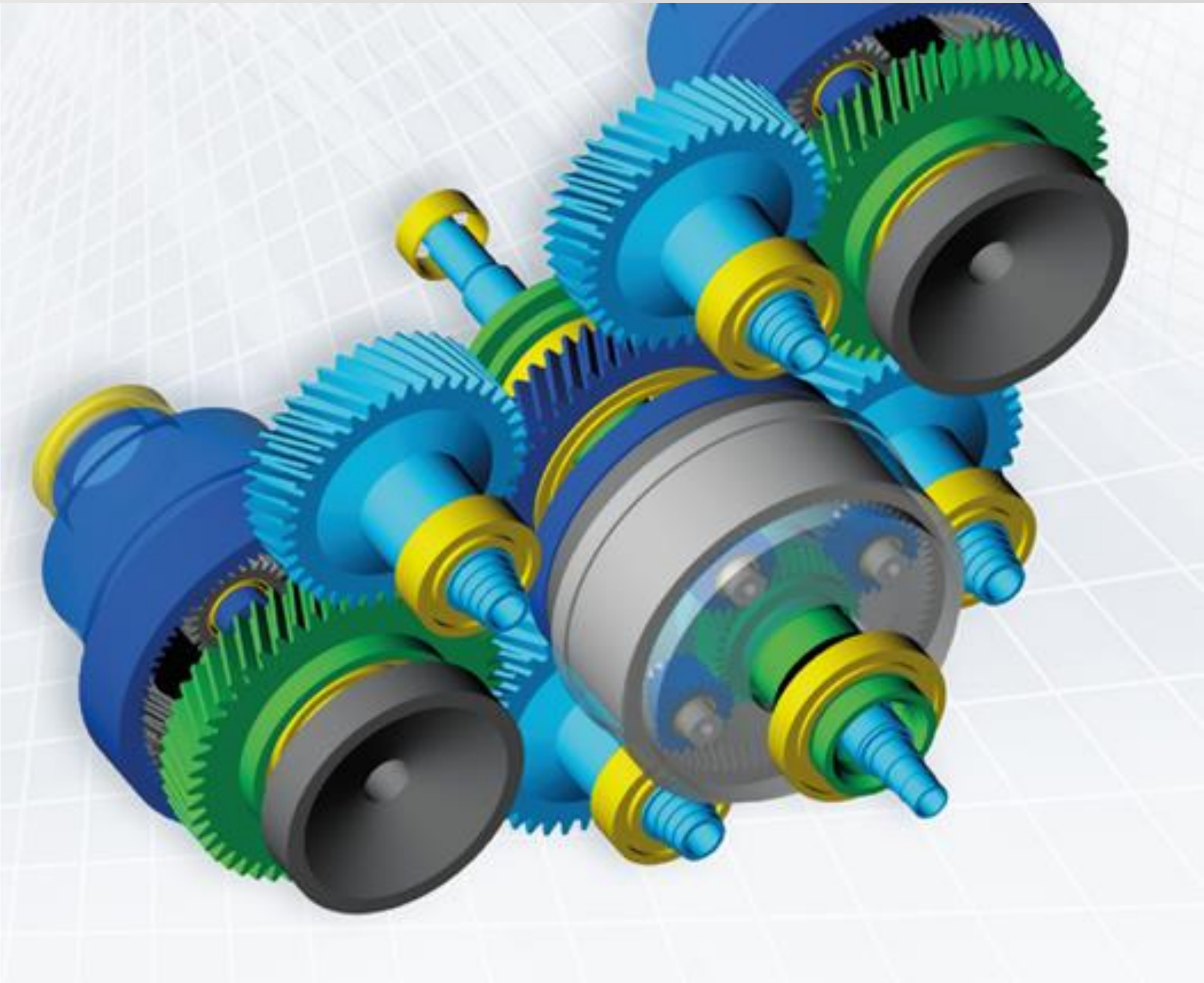


$$\frac{1}{2} P \delta_t = \frac{2.4\left(1-\nu^2\right)}{\pi E b} F_{bx}^2 + 0.294 \tan \alpha_F \frac{2.4\left(1-\nu^2\right)}{\pi E b} F_{by}^2$$

1. Asymmetric gears have benefits with respect to flank safety in case of mostly unidirectional load
2. With Langheinrich's adaption of W/B equations asymmetric gears can be considered in LTCA
3. Tooth stiffness of asymmetric gears are slightly higher due to differences in the definition of the fixpoint M
4. Root stress and stiffness calculation of asymmetric gears is reliable if certain peculiarities are kept in mind

Thank you for your attention!

This paper was first presented at International Conference on Gears 2019, 3rd International Conference on High Performance Plastic Gears 2019, 3rd International Conference on Gear Production 2019 in Garching/Munich.



Sharing Knowledge

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